Regularization-based Detection Algorithm for XL-MIMO Systems

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***Abstract*—The severe spatial correlation of near-field channel model imposes a great challenge upon the uplink signal detection in XL-MIMO systems. To overcome this detection obstacle, the optimized truncated singular value decomposition regularization detection (OTSVD) algorithm is firstly proposed to improve the detection performance in ill-conditioned channels, where an optimal parameter selection mechanism for OTSVD algorithm is given in terms of the mean squared error (MSE). Then, to achieve a better detection performance, the Tikhonov regularization detection (TRD) algorithm is also given. By well exploiting the diagonal dominance property of the channel matrix, a simple approach to determine the parameter in TRD is developed as well. Finally, simulation results are given to confirm the performance gains of these two proposed algorithms.**

***Index Terms*—MIMO detection, TSVD regularization, Tikhonov regularization, XL-MIMO, near-field.**

I. Introduction

Extremely large-scale multiple-input-multiple-output (XL-MIMO) is a promising technology to empower the next-generation communications [1], [2], [3]. However, with increased antenna array dimensions of the transmitter and receiver arrays, XL-MIMO pushes the electromagnetic wave operating region from far-field region to near-field one. Hence, several new channel characteristics need to be carefully considered, such as the near-field with non-uniform spherical wave and spatial channel non-stationarity [4], [5]. Unfortunately, these features render the channel ill-conditioned, imposing a big challenge on the signal detection for XL-MIMO.

Given the ill-conditioned channel matrix, the traditional linear detectors, such as zero forcing (ZF) and minimum mean square error (MMSE), may exhibit severe degradation because of the effect of noise amplification. Therefore, it is encouraged to reconstruct the channel matrix by low-rank approximation. By removing the small singular values, a better channel condition can be attained, which leads to an improved detection performance by surpassing the noise amplification in linear detections. Most importantly, although a certain of matrix information will be discarded, considerable performance gain still can be achieved by such low-rank approximations. To this end, a number of algorithms have been proposed [6], [7].

Specifically, a new detection algorithm that uses truncated singular value decomposition (TSVD) as preprocessing before lattice reduction (LR) is proposed in [6], which removes the least singular value of channel matrix $\mathbf{H}$. In [7], the analytical model for the post-detection signal-to-interference-plus-noise ratio (PDSINR) is derived. From the empirical cumulative distribution function (ECDF) of derived PDSINR, the truncated singular value is determined to achieve the comparable performance compared to MMSE.

In this paper, the near-field modeling of XL-MIMO communication systems is considered, which takes into account the phase and amplitude modeling of spherical waves. Based on this model, we introduce the optimized TSVD regularization detection (OTSVD) algorithm and give the criteria for selecting the truncation parameters. Additionally, we demonstrate that OTSVD outperforms the traditional linear detection algorithms in terms of MSE. To achieve more remarkable performance gain, we then propose the Tikhonov regularization detection (TRD) algorithm aiming at rectifying the ill-condition of channel matrix.

II. System Model

In this work, we consider an XL-MIMO system that the transmitter and receiver are equipped with K-element and N-element antenna arrays, respectively. In contrast to the far-field, the near-field channel depends on the distance between the users and the receiver. Without loss of generality, we assume that the users are uniformly distributed within the near-field range, and the basic uniform linear array (ULA) architecture with adjacent elements separated by distance $d = \lambda/2$ is considered. Considering these facts, the received signal vector $\mathbf{y} \in \mathbb{C}^N$ can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^K$ is transmitted symbol vector. The covariance matrix of $\mathbf{x}$ is $\mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{I}_K$, and $\mathbf{n} \in \mathbb{C}^N$ is the additive white Gaussian noise (AWGN) with covariance $\mathbb{E}\left[\mathbf{n}\mathbf{n}^H\right] = \sigma_n^2\mathbf{I}_N$ and zero mean. Let K represent the total number of transmitter antennas with N_t antennas at each user such that $N_t = K/N_k$. The overall channel matrix can be represented as

$$\mathbf{H} = [\mathbf{H}_1, \ldots, \mathbf{H}_{N_k}], \tag{2}$$

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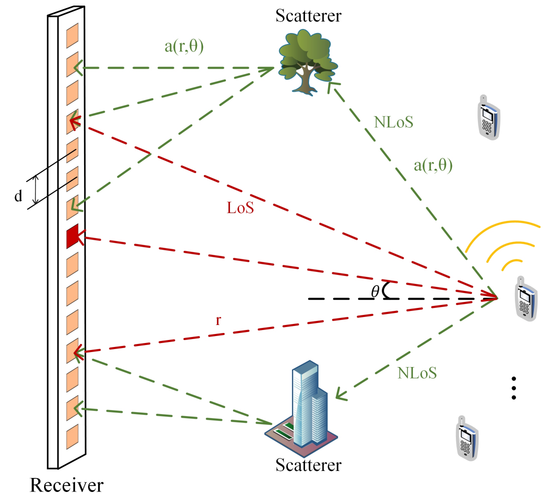

Fig. 1. The mixed LoS/NLoS near-field channel model for XL-MIMO.

where $\mathbf{H}_{n_k} \in \mathbb{C}^{N\times N_t}$ represents the sub-channel matrix corresponding to the n_k-th user which is defined by [8]

$$\mathbf{H}_{n_k} = \mathbf{H}_{\text{LoS}} + \mathbf{H}_{\text{NLoS}}. \tag{3}$$

Here, $\mathbf{H}_{\text{LoS}} \in \mathbb{C}^{N\times N_t}$ and $\mathbf{H}_{\text{NLoS}} \in \mathbb{C}^{N\times N_t}$ are the line-of-sight (LoS) and non-line-of-sight (NLoS) channels. To be more specific, the LoS channel matrix $\mathbf{H}_{\text{LOS}}$ is modelled as

$$\mathbf{H}_{\text{LoS}} = \mathbf{H}_{\text{LoS}}\left(r\right) = \frac{1}{r_{n,n_t}} e^{-j\frac{2\pi}{\lambda}r_{n,n_t}}, \tag{4}$$

where the r_{n,n_t} is the distance of the n-th antenna at receiver from the n_t-th antenna at each user. Meanwhile, the NLoS channel matrix $\mathbf{H}_{\text{NLOS}}$ can be described as

$$\mathbf{H}_{\text{NLoS}} = \frac{1}{\sqrt{L}}\sum_{l=1}^{L}\beta_l \mathbf{a}_R\left(r_l,\theta_l\right)\mathbf{a}_T^H\left(r_l,\theta_l\right), \tag{5}$$

where $\mathbf{a}_R\left(r_l,\theta_l\right)$ and $\mathbf{a}_T\left(r_l,\theta_l\right)$ denote the near-field receive and transmit array response vectors of ULA for non-uniform spherical wave (NUSW) model respectively, which are given as follows

$$\mathbf{a}_R(r,\theta) = \left[\sqrt{\frac{U_1}{U}}\frac{r}{r_1}e^{-j\frac{2\pi}{\lambda}(r_1)} \dots \sqrt{\frac{U_N}{U}}\frac{r}{r_N}e^{-j\frac{2\pi}{\lambda}(r_N)}\right]^T \tag{6}$$

with r is the distance between the center of transmitter's antenna array and the center of receiver's antenna array, and θ denotes the angle of departure (AoD) of the signal. The exact distance r_{n,n_t} in (4) presented by [8]

$$r_{n,n_t} = (nd - r\sin\theta)^2 + (r\cos\theta)^2 \overset{(a)}{=} r + nd\sin\theta + \frac{n^2d^2\cos^2\theta}{2r}. \tag{7}$$

The approximation (a) is derived from the second-order Taylor series expansion $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \mathcal{O}\left(x^3\right)$. In a rich scattering environment, i.e.,$(L \gg 1)$, the MIMO channel in (3) can achieve full rank due to the random phase-shifts imposed by scatters [4], then $L = 8$ is set in this paper. To our knowledge, this is the first time that signal detection is considered for such a near-field XL-MIMO scenario.

Given the system model in (1), to recover the transmitted signal $\mathbf{x}$, the traditional linear detection schemes like ZF and MMSE are commonly applied due to the significant detection performance, which work as follows

$$\widetilde{\mathbf{x}}_{\text{ZF}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{y}, \tag{8}$$

$$\widetilde{\mathbf{x}}_{\text{MMSE}} = \left(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_K\right)^{-1}\mathbf{H}^H\mathbf{y}, \tag{9}$$

respectively. Then, the signal detection $\hat{\mathbf{x}}$ is determined by quantizing $\hat{\mathbf{x}} = \lceil\widetilde{\mathbf{x}}_{\text{ZF}}\rfloor_{\mathcal{Q}} \in \mathcal{O}^K$ or $\hat{\mathbf{x}} = \lceil\widetilde{\mathbf{x}}_{\text{MMSE}}\rfloor_{\mathcal{Q}} \in \mathcal{O}^K$. However, due to the noise amplification, both of ZF and MMSE suffer from the ill-conditioned channel matrix a lot. In this condition, TSVD technique is applied to approximate the channel matrix by only retaining the first p largest singular values [9]. By doing this, considerable performance gain can be introduced with the improved channel condition. More specially, the rank-p approximation $\mathbf{H}_p$ of $\mathbf{H}$ is defined by

$$\mathbf{H}_p = \mathbf{U}\varSigma_p\mathbf{V}^H = \sum_{i=1}^{p}\sigma_i\mathbf{u}_i\mathbf{v}_i^H, \tag{10}$$

where $\varSigma_p = diag\left(\sigma_1,\dots,\sigma_p,0,\dots,0\right)$, $\mathbf{U}\in\mathbb{C}^{N\times N}$ and $\mathbf{V}\in\mathbb{C}^{K\times K}$ are orthogonal matrices, $p < \text{rank}(\mathbf{H})$ and the entries of the diagonal matrix $\varSigma_p$ with the smallest $K-p$ singular values of $\mathbf{H}$ replaced by zeros are ordered according to $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_p$. Intuitively, how to select a proper size of truncation parameter p for performance improvement is the key to TSVD, thus leaving an open question at this point.

III. Optimized TSVD Regularization Detection Algorithm

We now introduce an optimized choice rule of truncation parameter p to further improve the detection performance of TSVD for XL-MIMO systems, and this leads to the proposed OTSVD algorithm.

First of all, the mean square error (MSE) measures the discrepancy between an estimate and the corresponding true value, which directly accounts for the detection performance. Considering both bias and variance of the solution, the MSE of ZF is expressed as

$$\begin{aligned}\text{MSE}\left(\widetilde{\mathbf{x}}_{\text{ZF}}\right) &= E\left[\left(\widetilde{\mathbf{x}}_{\text{ZF}} - \mathbf{x}\right)^2\right] \\ &= E\left\{\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{n}\right]^H\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{n}\right]\right\} \\ &= \sigma_n^2\sum_{i=1}^{K}\frac{1}{\sigma_i^2}. \end{aligned}\tag{11}$$

On the other hand, the detection estimate $\widetilde{\mathbf{x}}_{\text{TSVD}}$ of TSVD can be written as follows

$$\begin{aligned}\widetilde{\mathbf{x}}_{\text{TSVD}} &= \left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{y} \\ &= \left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\left(\mathbf{H}\mathbf{x}+\mathbf{n}\right) \\ &= \left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{H}\mathbf{x} + \left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{n}. \end{aligned}\tag{12}$$

Given the low-rank approximation $\mathbf{H}_p$ in (10), $\mathbf{H}_p^H\mathbf{H}_p$ is a singular square matrix, for this reason the matrix inversion

Algorithm 1 OTSVD algorithm

Require: $\mathbf{H}$, $\mathbf{y}$.
Ensure: estimated transmit signal $\hat{\mathbf{x}}$
compute $\mathbf{H} = \mathbf{U}\mathit{\Sigma}\mathbf{V}^H$;
determine p according to (15) and (24);
compute rank-p approximation $\mathbf{H}_p$ of $\mathbf{H}$ according to (10);
compute $\widetilde{\mathbf{x}}$ according to (12);
output $\hat{\mathbf{x}} = \lceil \widetilde{\mathbf{x}} \rfloor_{\mathcal{Q}} \in \mathcal{O}^K$
end

operation in (8) is substituted with a pseudo-inverse. Based on it, a similar MSE of $\widetilde{\mathbf{x}}_{\text{TSVD}}$ can be obtained

$$
\begin{aligned}
&\text{MSE}\left(\widetilde{\mathbf{x}}_{\text{TSVD}}\right) = E\left[\left(\widetilde{\mathbf{x}}_{\text{TSVD}} - \mathbf{x}\right)^2\right] \\
&= E\Big\{\left[\left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{H}\mathbf{x} - \mathbf{x}\right]^H\left[\left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{H}\mathbf{x} - \mathbf{x}\right]\Big\} \\
&\quad + E\Big\{\left[\left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{n}\right]^H\left[\left(\mathbf{H}_p^H\mathbf{H}_p\right)^{\dagger}\mathbf{H}_p^H\mathbf{n}\right]\Big\} \\
&= E\Big\{\left[\left(\mathbf{H}_p^{\dagger}\mathbf{H} - \mathbf{I}\right)\mathbf{x}\right]^H\left[\left(\mathbf{H}_p^{\dagger}\mathbf{H} - \mathbf{I}\right)\mathbf{x}\right]\Big\} + E\left[\left(\mathbf{H}_p^{\dagger}\mathbf{n}\right)^H\left(\mathbf{H}_p^{\dagger}\mathbf{n}\right)\right] \\
&= \left\|\mathbf{H}_p^{\dagger}\mathbf{H} - \mathbf{I}\right\|_{\text{F}}^2 + \sigma_n^2\text{Tr}\left[\left(\mathbf{H}_p^{\dagger}\right)^H\mathbf{H}_p^{\dagger}\right] \\
&= \left\|\mathbf{V}\mathit{\Sigma}_p^{\dagger}\mathbf{U}^H\mathbf{U}\mathit{\Sigma}\mathbf{V}^H - \mathbf{I}\right\|_{\text{F}}^2 + \sigma_n^2\sum_{i=1}^{p}\frac{1}{\sigma_i^2} \\
&\overset{(b)}{=} (K-p) + \sigma_n^2\sum_{i=1}^{p}\frac{1}{\sigma_i^2},
\end{aligned} \tag{13}
$$

where the change in (b) holds because of the diagonal form $\mathit{\Sigma}_p^{\dagger} = diag\left(1/\sigma_1, \ldots, 1/\sigma_p, 0, \ldots, 0\right)$ and $\mathit{\Sigma} = diag\left(\sigma_1, \ldots, \sigma_p, \sigma_{p+1}, \ldots, \sigma_K\right)$. Based on (13), we want to find an optimized choice of truncation parameter p to minimize the MSE of OTSVD, i.e.,

$$
p^{\star} = \underset{1 \le p \le K}{\arg\min}\, \text{MSE}\left(\widetilde{\mathbf{x}}_{\text{TSVD}}\right). \tag{14}
$$

Theorem 1. *To solve the problem in* (14)*, the optimized choice of* $p^{\star}$ *should satisfy*

$$
\begin{aligned}
&p^{\star} = \max p, \\
&s.t. \sigma_n^2 < \sigma_p^2.
\end{aligned} \tag{15}
$$

Proof: To start with, for notational simplicity, let the right-hand side of (13) be denoted as the function $f(\cdot)$. It follows that

$$
f(i) = (K-i) + \frac{\sigma_n^2}{\sigma_1^2} + \ldots + \frac{\sigma_n^2}{\sigma_i^2}, \tag{16}
$$

where $i = 1, 2, \ldots, K-1$. Then, we arrive $f(i+1) < f(i)$ if

$$
\sigma_n^2 < \sigma_{i+1}^2. \tag{17}
$$

Hence, there is a value p satisfies the following conditions

$$
\sigma_n^2 < \sigma_i^2, \quad i = 1, \ldots, p \quad \text{and} \quad \sigma_n^2 > \sigma_{p+1}^2 \tag{18}
$$

such that

$$
f(p) < f(p-1) < \ldots < f(1), \tag{19}
$$

which corresponds to a descending order of $\text{MSE}\left(\hat{\mathbf{x}}_{\text{TSVD}}\right)$. Specifically, the parameter p that satisfies condition (18) is the desired one to minimize the $\text{MSE}\left(\widetilde{\mathbf{x}}_{\text{TSVD}}\right)$. ∎

Algorithm 2 TRD algorithm

Require: $\mathbf{H}$, $\mathbf{b} = \mathbf{H}^H\mathbf{y}$.
Ensure: estimated transmit signal $\hat{\mathbf{x}}$
compute $\mathbf{H} = \mathbf{U}\mathit{\Sigma}\mathbf{V}^H$;
determine Gram matrix $\mathbf{A}'$ according to (36)
$\widetilde{\mathbf{x}} = \left(\mathbf{A}'\right)^{-1}\mathbf{b}$;
output $\hat{\mathbf{x}} = \lceil \widetilde{\mathbf{x}} \rfloor_{\mathcal{Q}} \in \mathcal{O}^K$
end

Corollary 1. *Given the truncation parameter p selected according to (15), the proposed OTSVD algorithm achieves a better detection performance than ZF due to*

$$
\text{MSE}\left(\widetilde{\mathbf{x}}_{\text{TSVD}}\right) < \text{MSE}\left(\widetilde{\mathbf{x}}_{\text{ZF}}\right). \tag{20}
$$

Proof: According to the value p determined by (15), we have

$$
\sigma_n^2 > \sigma_i^2, \quad i = p+1, \ldots, K. \tag{21}
$$

Consequently, it follows

$$
\sum_{i=p+1}^{K}\frac{\sigma_n^2}{\sigma_i^2} > (K-p). \tag{22}
$$

After that, by simple calculation, we have

$$
\text{MSE}(\widetilde{\mathbf{x}}_{\text{TSVD}}) = (K-p) + \sigma_n^2\sum_{i=1}^{p}\frac{1}{\sigma_i^2} < \sigma_n^2\sum_{i=1}^{K}\frac{1}{\sigma_i^2} = \text{MSE}(\widetilde{\mathbf{x}}_{\text{ZF}}), \tag{23}
$$

which completes the proof. ∎

Here, we point out that the choice of parameter p is derived by a statistical way. In practice, for XL-MIMO systems with ill-conditioned channels, such a choice mechanism can be further relaxed to

$$
\sigma_p > 1 \quad \text{and} \quad \sigma_{p+1} < 1. \tag{24}
$$

Intuitively, this is easy to understand by removing all the small singular values that amplify the noise. By doing this, the effects of noise amplification can be effectively reduced.

IV. Tikhonov Regularization Detection Algorithm

When the TSVD regularization is more reliant with achieving significant performance gains in scenarios the channel matrix contains small singular values, Tikhonov regularization technique is also universally used to solve ill-conditioned problems. To find the regularized solution $\widetilde{\mathbf{x}}_{\lambda}$ of the equation in (1), the Tikhonov method is applied as

$$
\widetilde{\mathbf{x}}_{\lambda} = \left(\mathbf{H}^H\mathbf{H} + \lambda\mathbf{I}_K\right)^{-1}\mathbf{H}^H\mathbf{y}, \quad \lambda > 0. \tag{25}
$$

Here, the regularization parameter λ controls the "smoothness" of the regularized solution. If λ is too small, the numerical implementation may be unstable due to ill-conditioning of the original system. On the other hand, in the case of a large λ, the approximation error may become considerable [10]. Clearly, how to choose λ in a reasonable way is a key problem.

Considering the matrix $\mathbf{A} = \mathbf{H}^H\mathbf{H} = [a_{ij}] \in \mathbb{C}^{K \times K}$ with diagonal elements a_{jj} and off-diagonal elements $a_{ij}\,(i \neq j)$,

$\mathbf{A}' = \mathbf{H}^H\mathbf{H} + \lambda\mathbf{I} = [a'_{ij}]$ with diagonal elements a'_{jj} and off-diagonal elements a'_{ij} $(i \neq j)$ for notational simplicity, we have

$$a'_{jj} = a_{jj} + \lambda, \sum_{i\neq j}^{K} | a'_{ij} | = \sum_{i\neq j}^{K} | a_{ij} |, \quad j=1,2,\ldots,K. \tag{26}$$

Then, by letting

$$\alpha = \min_{1\leq j\leq K} \left\{ |a'_{jj}| - \sum_{i\neq j}^{K} |a'_{ij}| \right\}, \tag{27}$$

it is clear to see that matrix $\mathbf{A}'$ is diagonal dominant when $\alpha \geq 0$ holds [11]. To simplify the summation component on the right-hand side of (27), we assume that

$$m = \arg\max_{1\leq j\leq K} \sum_{i\neq j}^{K} |a'_{ij}|, \tag{28}$$

where m denotes the column with the maximum column sum norm of off-diagonal elements. Therefore, by considering the value of α in (27) when the column $j = m$, we have

$$\alpha = \min_{1\leq j\leq K} \left\{ |a'_{jj}| - \sum_{i\neq j}^{K} |a'_{ij}| \right\} \leq |a'_{mm}| - \sum_{i\neq m}^{K} |a'_{im}|. \tag{29}$$

Subsequently, by some manipulations, the following derivation can be achieved

$$\begin{aligned} \alpha &\leq |a'_{mm}| - \sum_{i\neq m}^{K} |a'_{im}| \\ &\overset{(c)}{\leq} |a_{mm}| + \lambda - \frac{1}{\sqrt{K}} \left| \sigma'_1 - \max_{1\leq j\leq K} |a'_{jj}| \right| \\ &\overset{(d)}{\leq} \max_{1\leq j\leq K} |a_{jj}| + \lambda - \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right|. \end{aligned} \tag{30}$$

Here, the transfer in (c) comes from the fact: given a general matrix $\mathbf{B} = [b_{ij}] \in \mathbb{C}^{K\times K}$, it satisfies [12]

$$\left| \sigma_1(\mathbf{B}) - \max_{1\leq j\leq K} |b_{jj}| \right| \leq \sqrt{K} \max_{1\leq j\leq K} \sum_{i\neq j}^{K} |b_{ij}|, \tag{31}$$

the inequality (d) holds due to $|a_{mm}| \leq \max_{1\leq j\leq K} |a_{jj}|$. Therefore, due to the fact $\alpha > 0$, the upper bound of α derived in (30) should be larger than 0, which results in

$$\lambda \geq \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right| - \max_{1\leq j\leq K} |a_{jj}| \tag{32}$$

to ensure the diagonal dominance property of matrix $\mathbf{A}'$. Next, to further fulfill the requirement of $\lambda > 0$ in Tikhonov regularization, we update the right-hand side of (32) via the absolute value, namely,

$$\lambda \geq \left| \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right| - \max_{1\leq j\leq K} |a_{jj}| \right|. \tag{33}$$

Furthermore, to determine the optimum value of λ, let

$$\lambda = \mu \left| \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right| - \max_{1\leq j\leq K} |a_{jj}| \right|, \quad \mu \geq 1 \tag{34}$$

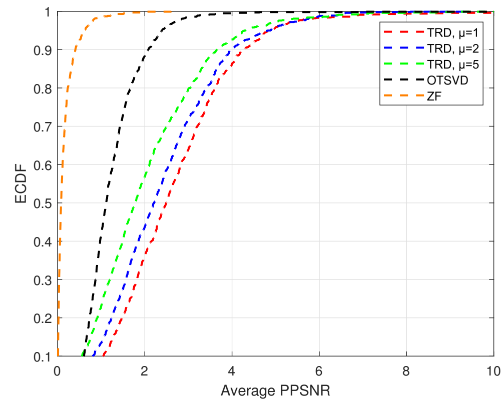

Fig. 2. ECDF of average PPSNR for the 512×64 XL-MIMO.

where μ is the proportionality constant. Then, the optimum value of λ is determined from the ECDF of post-processing signal-to-noise ratio (PPSNR). In particular, the PPSNR is a good indicator for the error rate performance, where the higher the probability of a large PPSNR accounts for the lower the uncoded error rate [13]. Moreover, the receiver can estimate the transmitted signal by applying the TRD detector $\mathbf{W}$ to the received signal, $\widetilde{\mathbf{x}} = \mathbf{W}\mathbf{y} = \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n}$, and $\mathbf{W}$ is derived as $\mathbf{W} = (\mathbf{A}')^{-1}\mathbf{H}^H$. Therefore, the PPSNR of i-th antenna is calculated as

$$\gamma_i = \frac{E_s \left|(\mathbf{W}\mathbf{H})_{i,i}\right|^2}{E_s \sum_{j\neq i} \left|(\mathbf{W}\mathbf{H})_{i,j}\right|^2 + \sigma_n^2 \left(\mathbf{W}\mathbf{W}^H\right)_{i,i}}, \tag{35}$$

where E_s denotes the signal energy at each transmit antenna. To illustrate it in a better way, the ECDF of PPSNR for ZF, OTSVD and TRD algorithm with different values of μ are plotted in Fig. 2. As can be seen clearly, for a 512×64 XL-MIMO system at SNR = 20dB with 4-QAM, greater performance is achieved with smaller value of μ. Specifically, there is a high likelihood of a large PPSNR when $\mu = 1$. Meanwhile, it is observed that the TRD performs better than ZF and OTSVD in terms of PPSNR.

In summary, based on the above analysis, the proposed TRD algorithm takes the form of Gram matrix

$$\mathbf{A}' = \mathbf{H}^H\mathbf{H} + \lambda\mathbf{I}, \tag{36}$$

where

$$\lambda = \left| \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right| - \max_{1\leq j\leq K} |a_{jj}| \right|. \tag{37}$$

This parameter λ mitigates the effect of noise amplification by enhancing the diagonal dominance property of the filter matrix. In addition, we point out that the impact of the noises has not been considered in the choice of λ in (37). The addition of noise guarantees convergence of the regularized solution [10], allowing it to adapt to the noise level, thus ensuring better performance in various scenarios. As a remedy solution, we update λ in the following way

$$\lambda_\sigma = \left| \frac{1}{\sqrt{K}} \left| \sigma_1 - \max_{1\leq j\leq K} |a_{jj}| \right| - \max_{1\leq j\leq K} |a_{jj}| \right| \sigma_n^2, \tag{38}$$

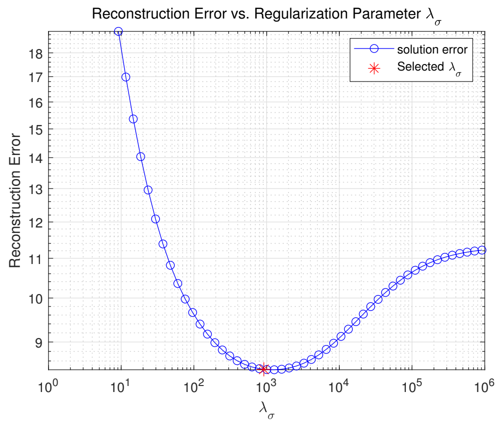

Fig. 3. Error versus parameter λ_σ for the 512×64 XL-MIMO.

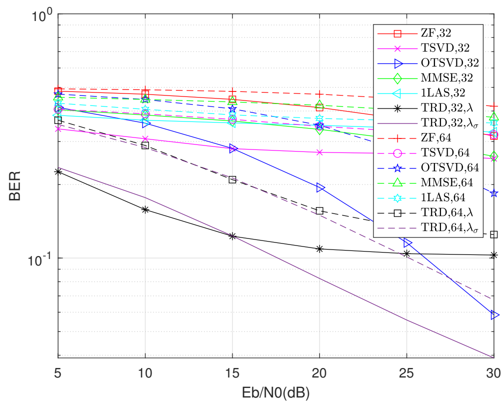

Fig. 4. BER versus SNR for the 512×64 and 512×32 XL-MIMO.

where the noise variance σ_n^2 serves as a scaling factor. According to Fig. 3, it can be verified that the TRD algorithm with λ_σ in (38) has the minimum error $\|\mathbf{x} - \widetilde{\mathbf{x}}_\lambda\|$ among the various λ_σ values. In fact, such a modification really facilitate the operations of proposed regularized scheme, which can be observed in simulations.

V. Simulation Results

In this section, the simulation results of the bit error rate (BER) performance against the signal-to-noise ratio (SNR) are provided for uplink near-field channel model in XL-MIMO systems. The carrier frequency is $f = 50GHz$. Each user has a ULA with eight antennas. The number of BS antennas is $N = 512$. Considering the antenna spacing is half-wavelength, the array aperture at BS can be calculated as $N \times \frac{c}{2f} = 512 \times \frac{0.003}{2} = 1.536m$.

Fig. 4 shows the BER performance comparison with respect to the proposed algorithms are illustrated in 512×64 and 512×32 XL-MIMO systems with 4-QAM. The legend “TSVD” refers to the algorithm proposed in [7]. Due to the ill-conditioned channel matrix, the performance of ZF and MMSE detectors are poor whereas the OTSVD and TRD algorithms lead to significant performance gains in both cases of 512×64 and 512×32 XL-MIMO. They also exhibit better performance compared to the TSVD algorithm. Moreover, the TRD algorithm with λ_σ achieves a faster convergence performance than that with λ, improving the atypical behaviour of the TRD with λ_σ at higher SNR, and both of them outperform other linear detection schemes. Simultaneously, they significantly improve the BER performance over the 1LAS detection with MMSE solution as initial solution. Performance limitation due to the intra-user interference in XL-MIMO systems with multiple antennas per user will be investigated in the future.

VI. Conclusion

In this paper, two regularization detection schemes are proposed to overcome the performance limitation in the near-field scenario of interest for the uplink detection of XL-MIMO systems. Compared to traditional linear detection schemes, the proposed OTSVD algorithm is able to achieve enhanced performance for ill-conditioned systems. Then, by analysis, a choice of the truncation parameter is determined in terms of MSE. We also proposed the TRD algorithm which outperforms OTSVD due to the more substantial PPSNR, and a simple way to obtain the parameter is described.

Acknowledgment

This work was supported in part by the National Key R&D Program of China under Grants No.2023YFC2205501, and in part by National Natural Science Foundation of China under Grants No.62371124.

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