Cluster-Based Low-Complexity Codebook Design for Hierarchical Beam Training in XL-MIMO

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***Abstract*—This paper proposes a cluster-based low-complexity codebook design scheme for hierarchical near-field beam training in extremely large-scale MIMO(XL-MIMO), referred to as the cluster hierarchical beam training (CHB). Specifically, to reduce the codebook dimensionality while preserving essential angle and distance information, the proposed CHB scheme employs a cluster-based approach to identify cluster centers as new polar-domain sampling points. By doing so, CHB significantly reduces the complexity of codeword generation. The generated codewords are then applied to hierarchical beam training, thereby further decreasing the associated training overhead. Finally, simulation results confirm that CHB reduces complexity while offering comparable or even superior performance to other codebook-based near-field beam training schemes.**

***Index Terms*—Extremely large-scale MIMO (XL-MIMO), near field communication, codebook design, beam training.**

I. INTRODUCTION

With the rapid development of wireless communication technologies, XL-MIMO has emerged as a critical technology for future 6G due to its high spatial multiplexing and spectral efficiency [1]. However, the transition in electromagnetic propagation characteristics from far-field planar waves to near-field spherical waves also renders traditional beamforming approaches unsuitable [2]. For this reason, much more effort has to be made for both codebook design and beam training as extra complexity and overhead are naturally incurred in the near-field scenarios.

Specifically, the polar-domain approach in [3] employs uniform angular sampling and non-uniform distance sampling to represent the near-field steering vector, but its large codebook dimension unfortunately requires exhaustive searching. The authors in [4] adopt a neural network to determine the optimal beam. However, the corresponding resource consumption remains proportional to the number of antennas, which imposes significant overhead in XL-MIMO systems. To further reduce the beam training overhead, a fast beam training scheme proposed in [5] employs a far-field codebook to determine candidate angles and then performs distance estimation. Meanwhile, the time-delay-based beam training approach in [6] leverages the near-field beam split effect over a wide bandwidth to control beams, leading to the optimal performance with extremely low training overhead. Nonetheless, the associated complex hardware architecture incurs high deployment costs and power consumption. Building on the the phase retrieval problem, [7] proposes a codebook design method based on GS iteration. Although the hierarchical beam training method in [7] achieves a favorable balance between high achievable rates and low training overhead, the matrix pseudoinverse calculation required for codebook design remains computationally expensive.

In this paper, to reduce the codeword design complexity and beam training overhead, we propose a novel low-complexity scheme based on the concepts of clustering and hierarchical beam training, which is named as cluster hierarchical beam training (CHB). In particular, by employing clustering to determine new angle and distance sampling points in the polar domain, the dimensionality of the codebook is significantly reduced, thereby reducing the computational complexity of codeword generation. Subsequently, the dimension-reduced codebook is applied to hierarchical beam training to further reduce the training overhead. Finally, we highlight the advantages of CHB in terms of complexity and demonstrate its superior performance through simulation results.

II. SYSTEM MODEL

As shown in Fig. 1, considering the typical downlink beam training of a narrow-band XL-MIMO communication system, base station (BS) is equipped with a uniform linear array (ULA) comprising N antennas for communication with a single-antenna uesr equipment (UE).

When the distance between BS and UE is smaller than the Rayleigh distance $Z = \frac{2D^2}{\lambda}$ with D and λ denoting the antenna array aperture and the carrier wavelength respectively, UE operates in the near-field region of the electromagnetic radiation [8]. Consequently, the traditional far-field plane-wave assumption no longer holds, and a near-field channel can be characterized by a spherical-wave model:

$$\mathbf{h} = \sqrt{N} g_l \mathbf{b}\left(\theta, r\right), \tag{1}$$

where g_l is the complex-valued channel gain. Here, $\mathbf{b}\left(\theta, r\right)$ represents the near-field steering vector:

$$\mathbf{b}\left(\theta, r\right) = \frac{1}{\sqrt{N}} \left[e^{-j\frac{2\pi}{\lambda}\left(r^{(1)}-r\right)}, ..., e^{-j\frac{2\pi}{\lambda}\left(r^{(N)}-r\right)}\right]^H, \tag{2}$$

where θ indicates the spatial angle at BS, and r represents the distance from UE to the center of the antenna array. The distance between the n-th antenna at BS and UE is given by $r^{(n)} = \sqrt{r^2 + \delta_n^2 d^2 - 2r\delta_n d\theta}$, where $d = \frac{\lambda}{2}$ is the antenna spacing and $\delta_n = \frac{2n-N-1}{2}$ with $n = 0, 1, 2, ..., N$. Under the downlink near-field channel model, the received signal at UE is expressed as:

$$y = \mathbf{h}^H \mathbf{v} s + n_0 = \sqrt{N} g_l \mathbf{b}^H\left(\theta, r\right) \mathbf{v} s + n_0, \tag{3}$$

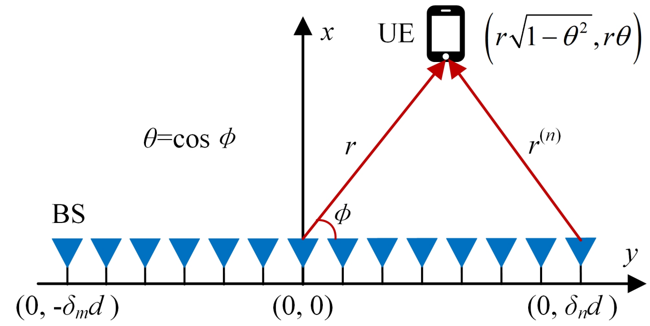

Fig. 1. A downlink near-field XL-MIMO communication system.

where $\mathbf{v} \in \mathbb{C}^{N\times 1}$ indicates the beamforming vector at BS, $s \in \mathbb{C}$ represents the transmitted symbol and n_0 is the received additive white Gaussian noise (AWGN) component with power σ^2. As such, given perfect CSI information (θ, r), the optimal XL-MIMO BS beamforming vector can be obtained as $\mathbf{v}^* = \mathbf{b}(\theta, r)$ [3].

The design principle of the codebook is to cover the largest possible spatial range, enabling precise beam alignment in different directions. The current effective approach is to construct the polar-domain codebook $\mathbf{M}$ by including a large number of the optimal beamforming vectors from different sampling points (θ_n, r_n^s):

$$\mathbf{M} = \left[\mathbf{b}\left(\theta_1, r_1^1\right), ..., \mathbf{b}\left(\theta_N, r_N^1\right), ..., \mathbf{b}\left(\theta_N, r_N^{S_N}\right)\right], \quad (4)$$

where N represents the number of antennas, and S_N denotes the number of distance rings in polar sampling. The angle θ is uniformly sampled as $\theta_n = \frac{2n-N+1}{N}, n = 0, 1, ..., N$, while the distance r is non-uniformly sampled with a threshold distance Z_Δ [3] according to formula $r_n^s = \frac{1}{s} Z_\Delta \left(1-\theta_n^2\right), s = 1, 2, ..., S_N$. Here, n indicates the different angle directions used for sampling, while s indicates the various rings used for distance sampling. To be specific, each column of $\mathbf{M}$ represents a codeword $\mathbf{w}_{s,n} = \mathbf{b}\left(\theta_n, r_n^s\right)$, and the purpose of the exhaustive search in near-field beam training is to find the optimal codeword to serve as the beamforming vector $\mathbf{v}$ in the polar-domain codewords that maximizes the received signal power. However, since the dimension of a polar-domain codebook is $N \times S_N$, exhaustive search incurs substantial beam training overhead particularly in XL-MIMO systems.

In order to reduce the beam training overhead while maintaining an acceptable performance loss in achievable rate, the GS-based codeword design scheme in [7] is proposed. To evaluate the effectiveness of a given codeword $\mathbf{w}$, the concept of beamforming gain is introduced:

$$G(\mathbf{w}, \theta, r) = \sqrt{N}\mathbf{b}(\theta, r)^H \mathbf{w}. \quad (5)$$

According to the polar-domain codebook in (4), the beam pattern obtained by the codeword $\mathbf{w}$ can be simplified to $\mathbf{M}^H\mathbf{w}$, which is composed of the beamforming gain at various spatial positions. Moreover, the ideal beam pattern of the codeword $\mathbf{w}$ is defined as:

$$\mathbf{g_w} = \left[g_\mathbf{w}(\theta_1, r_1^1), \cdots, g_\mathbf{w}(\theta_N, r_N^1), \cdots, g_\mathbf{w}(\theta_N, r_N^{S_N})\right], \quad (6)$$

where $g_\mathbf{w}(\theta, r) = |g_\mathbf{w}(\theta, r)|e^{jf_\mathbf{w}(\theta, r)}$ is the theoretical beamforming gain. The amplitude information $|g_\mathbf{w}(\theta, r)|$ in target angle coverage and distance coverage are fixed and flattened, while the phase information $f_\mathbf{w}(\theta, r)$ can be designed flexibly.

To ensure an acceptable performance loss, the GS-based codeword design scheme in [7] seeks to maximize the achievable rate of communication systems by minimizing the discrepancy between the ideal beam pattern and the beam pattern obtained by $\mathbf{w}$, namely:

$$\min_{\mathbf{w}, f_\mathbf{w}(\theta, r)} \left\|\mathbf{M}^H\mathbf{w} - \mathbf{g_w}\right\|_2^2. \quad (7)$$

In particular, the GS iteration algorithm starts with a randomly selected initial codeword $\mathbf{w}^{(0)}$. Under the constraint of constant amplitude, the two variables $\mathbf{w}$ and $\mathbf{g_w}$ are iteratively updated:

$$\mathbf{g}_\mathbf{w}^{(s)} = \mathbf{M}^H\mathbf{w}^{(s)}, \mathbf{w}^{(s+1)} = \left(\mathbf{M}\mathbf{M}^H\right)^{-1}\mathbf{M}\mathbf{g}_\mathbf{w}^{(s)}, \quad (8)$$

where (s) represents the iteration index. However, the computational cost of calculating the pseudoinverse of matrix $\mathbf{M}$ during the iteration is rather expensive, raising questions about the low-complexity feasibility of codeword generation.

III. Proposed Clustering Based Hierarchical Near-Field Codebook Design

This section begins by introducing the objectives of codeword design in the proposed CHB scheme. After that, the CHB algorithm is proposed, where the complexity comparison with other codebook-based schemes is also provided.

A. Cluster-Based Hierarchical Near-Field Codebook Design

In principle, the proposed CHB scheme seeks to maximize the achievable rate by minimizing the discrepancy between the designed codebook $\mathbf{W}$ and the polar-domain codebook $\mathbf{M}$. Specifically, $\mathbf{W}$ contains cluster centers μ^θ and μ^r in the angle and distance domains, while $\mathbf{M}$ comprises the sampling points (θ, r) introduced in section II. To approximately evaluate the discrepancy between these two codebooks, we define the sum of the squared distances between all sampling points and their corresponding cluster centers in $\mathbf{M}$ as the distance $d^2(\mathbf{W}, \mathbf{M})$ between $\mathbf{W}$ and $\mathbf{M}$, expressed as:

$$d^2(\mathbf{W}, \mathbf{M}) = \sum_{\theta, r \in \mathbf{M}} \min_{\mu^\theta, \mu^r \in \mathbf{W}} \left(\| \theta - \mu^\theta \|^2 + \| r - \mu^r \|^2\right). \quad (9)$$

By grouping numerous high-dimensional data points into a limited number of cluster centers, the reduction of the data dimensionality in a codebook can be achieved. However, directly minimizing such a distance function is highly complex. By separately reducing angular and distance variations to generate cluster centers, the CHB approximates the minimization of the discrepancy between $\mathbf{W}$ and $\mathbf{M}$.

a) Angle and Distance Clustering: The objective is to perform k_θ-class clustering for the angle θ and k_r-class clustering for the distance r, which is accomplished by iteratively minimizing the following two cost functions:

$$J_\theta = \sum_{i=1}^{N} \sum_{j=1}^{k_\theta} \left(c_i^\theta = j\right) \left\|\theta_i - \mu_j^\theta\right\|^2 \tag{10a}$$

$$J_r = \sum_{i=1}^{S_N} \sum_{j=1}^{k_r} \left(c_i^r = j\right) \left\|r_i - \mu_j^r\right\|^2, \tag{10b}$$

where θ_i represents the angular sampled points and r_i indicates the sampled distance rings in $\mathbf{M}$. Notably, μ_j^θ and μ_j^r represent the j-th cluster center in angular and distance respectively, and c_i denotes the class label to which the data point belongs.

In this clustering procedure, angle clustering starts by assigning each angular sampling point θ_i to the nearest center:

$$c_i^\theta = \underset{j}{\operatorname{argmin}} ||\theta_i - \mu_j^\theta||^2, \tag{11}$$

and then updates the cluster centers according to:

$$\mu_j^\theta = \frac{1}{n_j} \sum_{c_i^\theta = j} \theta_i, \tag{12}$$

where n_j represents the number of data points belonging to the j-th class. By repeatedly iterating these steps until the error function J_θ falls below a threshold ϵ, fewer angular cluster centers are produced. Similarly, the centers for distance clustering are determined in an analogous procedure. After clustering, both the angular and distance cluster centers are sorted in an ascending order, resulting in the corresponding sets: $\mathbf{W}_\theta = \{\mu_1^\theta, \mu_2^\theta, \cdots, \mu_{k_\theta}^\theta\}$ and $\mathbf{W_r} = \{\mu_1^r, \mu_2^r, \cdots, \mu_{k_r}^r\}$. These cluster centers are subsequently employed to determine the coverage ranges and generate codewords.

b) Calculation of Beam Codeword Coverage: In cluster-based near-field codeword design, each beam codeword covers more than a single angle-distance point, spanning a coverage region in the polar domain. To construct a multi-resolution hierarchical codebook, angular and distance cluster centers are initially paired to form all possible combinations $\left(\mu_m^\theta, \mu_n^r\right)$, where $m \in \{1, 2, ..., k_\theta\}, n \in \{1, 2, ..., k_r\}$. The coverage ranges of these centers are then computed and recorded for each codeword. Specifically, to determine the angle coverage range of the center $\left(\mu_m^\theta, \mu_n^r\right)$, the procedure first calculates the distance between each angle sampling point θ_i and the cluster center:

$$d_m^{\theta_i} = |\theta_i - \mu_m^\theta|, i = 1, 2..., N. \tag{13}$$

After computing these distances, the algorithm sorts the angular sampling points in ascending order. This produces an ordered sequence of angle indices $\{\theta_{m1}, \theta_{m2}, ..., \theta_{mN}\}$, where $d_m^{\theta_{m1}} < d_m^{\theta_{m2}} < ... < d_m^{\theta_{mN}}$. Then, the algorithm selects k angle samples with the smallest distances to define the coverage range for the current cluster μ_m^θ:

$$\begin{aligned} \theta_{\min,m} &= \min\{\theta_{m1}, \theta_{m2}, ..., \theta_{mk}\}, \\ \theta_{\max,m} &= \max\{\theta_{m1}, \theta_{m2}, ..., \theta_{mk}\} \end{aligned} \tag{14}$$

with $k = \frac{N}{k_\theta}$.

Algorithm 1 Cluster-based codeword design

Input: $\mathbf{M}$, $\{\theta_1, \theta_2, ..., \theta_N\}$, $\{r_1, r_2, ..., r_{S_N}\}$, k_θ, k_r

1: Initialization: Randomly generate the angle and distance cluster centers $\left\{\mu_1^\theta, \mu_2^\theta, ..., \mu_{k_\theta}^\theta\right\}$ and $\left\{\mu_1^r, \mu_2^r, ..., \mu_{k_r}^r\right\}$
2: **for** $i = 1, 2, ..., N$ **do**
3: assign θ_i to the nearest center by (11)
4: update the cluster centers $\left\{\mu_1^\theta, \mu_2^\theta, ..., \mu_{k_\theta}^\theta\right\}$ by (12)
5: **end for**
6: Jump to Step 2 until $J_\theta < \epsilon$
7: sort $\left\{\mu_1^\theta, \mu_2^\theta, ..., \mu_{k_\theta}^\theta\right\}$ in ascending order to get $\mathbf{W}_\theta$
8: obtain $\mathbf{W_r} = \left\{\mu_1^r, \mu_2^r, ..., \mu_{k_r}^r\right\}$ in the same steps
9: pair all possible cluster center combinations: $\left(\mu_m^\theta, \mu_n^r\right)$
10: **for** $m = 1, 2, ..., k_\theta$ **do**
11: **for** $n = 1, 2, ..., k_r$ **do**
12: calculate the near-field codeword $\mathbf{w}_{mn}$ by (2)
13: **end for**
14: **end for**
15: generate codebook $\mathbf{W} = [\mathbf{w}_{11}, \mathbf{w}_{12}, ..., \mathbf{w}_{mn}, ..., \mathbf{w}_{k_\theta k_r}]$
16: **for** $m = 1, 2, ..., k_\theta$ **do**
17: calculate $d_m^{\theta_i}$ from θ_i to μ_m^θ by (13) and sort θ_i
18: find the angle coverage $(\theta_{\min,m}, \theta_{\max,m})$ of μ_m^θ by (14)
19: **end for**
20: get distance coverage $(r_{\min,n}, r_{\max,n})$ in the same steps
21: combine $\mathbf{B}_{mn} = [\theta_{\min,m},\ \theta_{\max,m},\ r_{\min,n},\ r_{\max,n}]$
22: obtain the beam coverage $\mathbf{B_c}$ by (15)

Output: Codebook $\mathbf{W}$ and beam coverage $\mathbf{B_c}$

The upper and lower bounds of the coverage range are respectively defined as the maximum and minimum values of these k angle samples: $(\theta_{\min,m}, \theta_{\max,m})$. Similarly, the distance coverage range $(r_{\min,n}, r_{\max,n})$ is obtained by the same approach, resulting in a complete beam coverage range $\mathbf{B_c}$ that serves as the codeword record:

$$\mathbf{B_c} = [\mathbf{B}_{11}, \mathbf{B}_{12}, ..., \mathbf{B}_{mn}, ..., \mathbf{B}_{k_\theta k_r}], \tag{15}$$

where $\mathbf{B}_{mn} = [\theta_{\min,m},\ \theta_{\max,m},\ r_{\min,n},\ r_{\max,n}]$. Next, for each pair of cluster centers $\left(\mu_m^\theta, \mu_n^r\right)$, the near-field steering vector $\mathbf{b}\left(\mu_m^\theta, \mu_n^r\right)$ is computed via (2) to serve as the near-field codeword $\mathbf{w}_{mn}$. Finally, these codewords are combined to form a cluster-based hierarchical near-field codebook $\mathbf{W}$ of size $k_\theta \times k_r$. Algorithm 1 outlines the generation of near-field codewords and their respective coverage ranges via clustering. By varying the number of cluster centers, the algorithm constructs multiple multi-resolution codebooks, further reducing beam training overhead when applied to hierarchical beam training.

c) Hierarchical Near-Field Beam Training: Hierarchical codebooks are widely used in conventional massive MIMO systems to reduce training overhead by progressively narrowing the search space through multi-resolution codebooks [9]. Here, we extend this concept to near-field beam training in XL-MIMO, with the critical distinction lying in the transition from one-dimensional angle-domain searching to two-dimensional polar-domain joint searching.

As illustrated in Fig. 2, the hierarchical codebook spans different angle and distance ranges, with the coverage of each codeword discussed in the preceding sections. Lower-layer

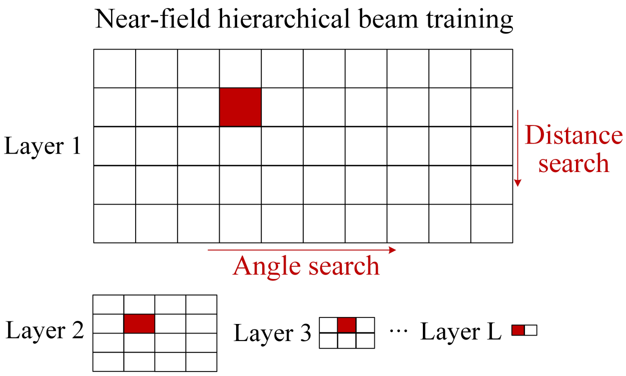

Fig. 2. Near-field hierarchical beam training process.

codebooks adopt coarser resolution and lower dimensionality, whereas upper-layer codebooks refine the search area by increasing both resolution and dimensionality. In the proposed scheme, we define L layers of subcodebooks. In the l-th layer, the angle and distance coverage are given by $[\theta_{\min}^{(l)}, \theta_{\max}^{(l)}]$ and $[r_{\min}^{(l)}, r_{\max}^{(l)}]$, and the step sizes $\Delta\theta^{(l)}$ and $\Delta r^{(l)}$ are determined by the number of cluster centers $k_\theta^{(l)}$ and $k_r^{(l)}$:

$$\Delta\theta^{(l)} = \frac{\theta_{\max}^{(l)} - \theta_{\min}^{(l)}}{k_\theta^{(l)}}, \Delta r^{(l)} = \frac{r_{\max}^{(l)} - r_{\min}^{(l)}}{k_r^{(l)}}. \quad (16)$$

A larger step size corresponds to lower resolution and is typically employed in the initial layers to enable broad but efficient scanning. As the layer index increases, a simple and effective strategy is to double the number of cluster centers (e.g. $k_\theta^{(l+1)} = 2k_\theta^{(l)}$), thereby halving the step size and refining the search region. For instance, a typical three-layer codebook might begin with coarse-grained clustering (e.g. $k_\theta^{(1)} = 64$ and $k_r^{(1)} = 4$) to cover the full angular range $\theta \in [-1, 1]$ and a predefined distance range, and then progressively narrow those ranges in subsequent layers. Notably, the number of angular and distance cluster centers can be flexibly adjusted to accommodate dynamic XL-MIMO scenarios [11].

In practice, using low-resolution codebooks in the initial layer to globally scan the received signal power substantially reduces the pool of candidate regions. Subsequent codebook searches leverage the prior information gleaned from the lower layer, retaining only the codewords that fall within the high-gain candidate areas. Consequently, the number of codewords to be scanned is much smaller than the size of the codebook, leading to significantly reduced beam training overhead. Compared with exhaustive search methods, this approach achieves a balanced trade-off between performance and overhead.

B. Complexity Analysis and Comparison

In general, the complexity of codeword design is primarily determined by the codeword-generation process. Meanwhile, the beam training overhead is chiefly dictated by the size of the codebook. In the GS-based iterative near-field 2D hierarchical codebook training scheme [7], the codeword design involves both the ideal and practical codeword design stages. Each stage relies on matrix pseudoinverse computations. Specifically, the ideal phase computes the pseudoinverse of $\mathbf{M}$ with complexity $O(NS^2)$ per codeword, while the practical phase employs alternating analog and digital processing matrix optimizations, incurring $O(NN_{RF}^2 + NL_\theta)$. Simply, the total complexity is approximately $O(NN_{RF}^2)$, where N_{RF} is the number of radio frequency (RF) chains. Thus, a single codeword requires $O(NS^2 + NN_{RF}^2)$, yielding $O(N^2S^2 + N^2N_{RF}^2)$ overall. In contrast, the CHB focuses solely on two clustering processes, which requires $O(Nk_\theta + Sk_r)$ to cluster angles and distances, approximately two orders of magnitude lower than the GS-based scheme.

Regarding beam training overhead, the exhaustive scheme needs to traverse the entire polar-domain codebook, incurring overhead of $O(NS)$. From the hierarchical beam training perspective, the overhead is dictated by the initial low-resolution search. Therefore, the beam training overhead of CHB is $O\left(\sum\left(k_\theta^{(l)} k_r^{(l)}\right)\right)$, where $k_\theta^{(l)} k_r^{(l)}$ denotes the number of codewords to be searched at layer l. In many cases, this overhead can be approximated by only the first layer $O\left(k_\theta^{(1)} k_r^{(1)}\right)$. Since the low-resolution codebook has a smaller dimension, the hierarchical approach significantly reduces training overhead compared with exhaustive search.

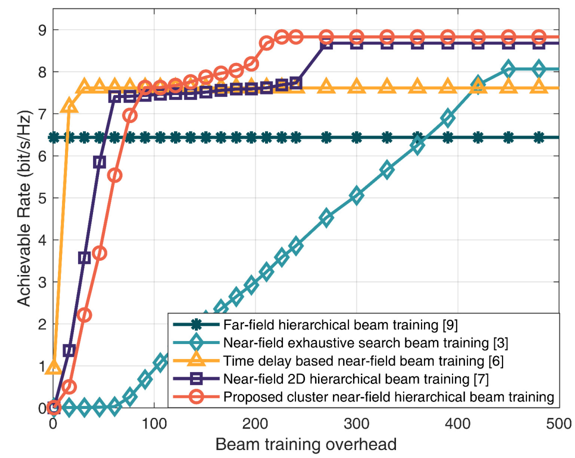

Fig. 3. Achievable sum-rate performance comparison versus overhead.

IV. Simulation Results

In this section, we present simulation results to evaluate the effectiveness of the proposed CHB scheme. We consider a narrowband XL-MIMO system with 512 antennas at BS. The system bandwidth is 100 MHz, and the carrier frequency is 50 GHz, which corresponds to a wavelength of 0.006 meters. Since the antenna spacing is half of the wavelength, the array aperture is $D = N \times \frac{\lambda}{2} = 1.536m$. The calculated Rayleigh distance is $Z = \frac{2D^2}{\lambda} = 786.432m$. In our simulations, the user distance ranges from $(20, 100)$, which is well within the Rayleigh distance, ensuring that all users operate in the near-field regime. Additionally, the angle range is $\theta \sim \mathcal{U}(-1, 1)$, and the complex path gain g_l satisfies $g_l \sim \mathcal{CN}(0, 1)$.

Fig. 3 shows the achievable rate versus beam training overhead for various schemes in XL-MIMO systems. For

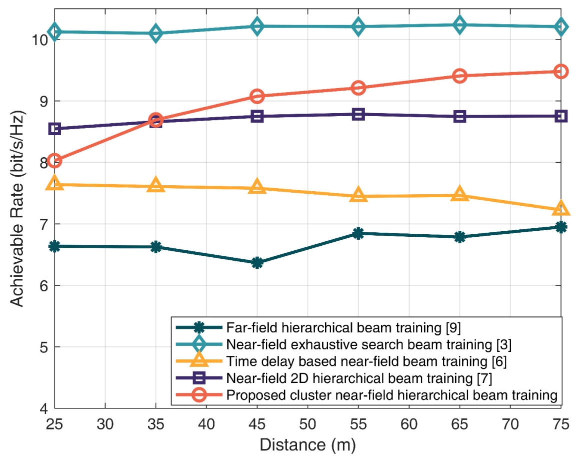

Fig. 4. Achievable sum-rate performance comparison versus distance.

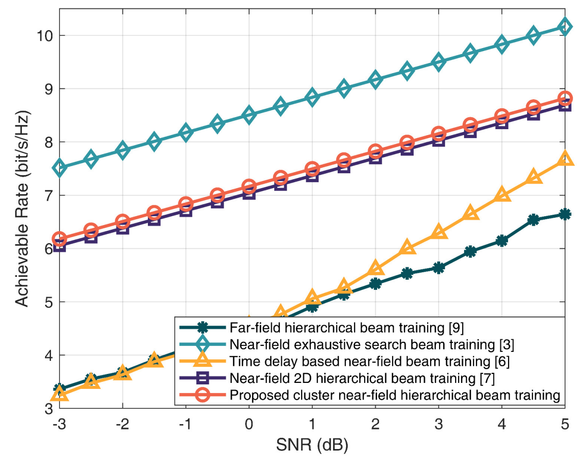

Fig. 5. Achievable sum-rate performance comparison versus SNR.

our simulations, we employ a three-layer multi-resolution codebook. The size of the first-layer codebook is designed to be 192, while the second and third layers require scanning 32 and 3 codewords, resulting in a total beam training overhead of 227. Clearly, the CHB scheme outperforms the legacy DFT codebook [9], [10]. Although near-field exhaustive search achieves optimal performance as the training overhead increases [3], it is constrained by the enormous codebook size of an overhead $512 \times 16 = 8192$. The time-delay scheme [6] requires a beam training overhead of only 16. However, it faces multiple hardware constraints in the narrowband system in this paper. The near-field 2D hierarchical beam training also employs a multi-resolution codebook design [7], incurring a training overhead of $256 + 8 + 4 = 268$. Although this method strikes a reasonable balance between overhead and rate, its high codeword-generation complexity hinders dynamic adjustments of each layer's codebook. By contrast, the CHB scheme adapts codebook sizes to channel conditions, thereby surpassing the near-field 2D scheme in both overhead and achievable rate.

Fig. 4 illustrates the impact of the distance from UE to BS on the beam training performance. As shown, the proposed CHB scheme performs closer to that of the optimal exhaustive search. Moreover, at 65 meters, it enhances performance by more than 10% compared with the near-field 2D scheme. Fig. 5 depicts the impact of system SNR on the achievable rate, demonstrating that the CHB scheme remains near-optimal, with only a small gap relative to the near-field 2D approach. When considering codeword-generation complexity, the CHB delivers superior overall performance.

V. Conclusion

In this paper, we propose a low-complexity codeword generation scheme CHB based on the clustering concept. The beam coverage range corresponding to each codeword is calculated, enabling its application in hierarchical near-field beam training to further reduce training overhead. Notably, this scheme achieves low complexity and overhead in codeword design and beam training respectively, facilitating system initialization, maintenance, and real-time responsiveness in near-field scenarios. Simulation results demonstrate that the proposed codeword generation scheme delivers high-quality beam training performance in near-field systems.

Acknowledgment

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