Smooth Penalty Detection Methods for Quantized and Distributed Massive MIMO Systems

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***Abstract*—In this paper, we propose two efficient detectors for massive MIMO systems equipped with low-resolution analog-to-digital converters (ADCs) under a distributed architecture. Specifically, we first reformulate the discrete maximum likelihood (ML) detection as a continuous penalized problem, thereby enabling the use of continuous optimization techniques. Then, to improve computational efficiency, we replace the original non-smooth penalty with a smooth surrogate and develop a distributed homotopy penalty detection (HPD) algorithm to solve the resulting problem. For enhanced detection accuracy, we propose a general multi-term smoothing framework that uses multiple smooth terms to approximate the non-smooth penalty. By theoretical analysis, we prove that the approximation error decreases monotonically as the number of smoothing terms increases, such that the optimum of smooth penalized problem converges to the ML solution. Based on it, we introduce a fusion penalty model and unfold its iterative solver into a deep neural network (DNN), termed FPD-Net, to further improve convergence and detection performance. Numerical results demonstrate that the proposed HPD algorithm achieves significant BER gains compared to conventional detectors, while FPD-Net provides additional performance improvements and faster convergence.**

***Index Terms*—Massive MIMO system, low-resolution ADCs, distributed signal detection, deep learning.**

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is a key technology for enhancing the capacity and spectral efficiency of B5G and 6G wireless communication systems [1]. However, scaling up the number of antennas at the base station (BS) significantly increases power consumption, especially for analog-to-digital converters (ADCs), whose power grows exponentially with the quantization bits. To mitigate these costs, low-resolution ADCs, e.g., 1–3 bits, have emerged as an attractive solution [2]–[5]. In such quantized systems, the Bussgang-based linear approximation is widely adopted to model the quantization, where the distortion is treated as additive white Gaussian noise (AWGN) that is uncorrelated with the quantizer output. Based on this model, the Bussgang-based minimum mean-squared error (BMMSE) has been extensively studied [2]. To further reduce complexity, the gradient descent (GD) [3] and Newton method (NM) [4] have also been employed. Unfortunately, the performance of these methods still remains limited, as they ignore the discrete-constellation constraint of the transmitted symbols.

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Besides, a more challenging problem of massive MIMO system also results from transferring the dramatic increase of raw data for signal processing [6]–[8]. To this end, decentralized baseband processing (DBP) architecture partitions the BS antennas into multiple distributed units (DUs), thereby reducing the bandwidth and distributing the complexity [6]. Based on DBP, distributed detection techniques such as conjugate gradient (CG) and alternating direction method of multipliers (ADMM) have been developed [7]. However, as they are quantization-unaware, their performance degrades significantly when applied to systems with low-resolution ADCs.

In this paper, we focus on massive MIMO detection with low-resolution ADCs in a distributed setting. By transforming discrete ML detection into a continuous formulation, we establish an explicit equivalence condition for multi-bit quantization. Since the resulting penalty is non-smooth, we construct an L^2-optimal smooth surrogate and develop a distributed HPD algorithm for efficient computation. We further propose a multi-term smoothing scheme that approximates the non-smooth penalty with a sum of smooth terms, and show that the approximation error vanishes and the resulting solutions converge to the ML optimum as the number of terms grows. Based on it, we develop a fusion penalty model and construct the distributed FPD-Net detector, where the parameters are learned via a hyper-network, to accelerate convergence and enhance performance. To the best of our knowledge, this is the first work that jointly considers low-resolution quantization and distributed massive MIMO detection.

II. SYSTEM MODEL

A. System Model with Low-Resolution ADCs

We consider the massive MIMO system, with N_t transmit and N_r receive antennas. Let $\bar{\mathbf{x}} \in \mathcal{O}^{N_t}$ denote the transmitted vector from M^2-quadrature amplitude modulation (QAM) constellation, and the received signal is given by

$$\bar{\mathbf{y}} = \mathcal{Q}_b(\bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}}). \tag{1}$$

Here, $\bar{\mathbf{H}} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix, $\bar{\mathbf{n}} \in \mathbb{C}^{N_r}$ is the AWGN vector with zero mean and covariance $\sigma_n^2\mathbf{I}$, and $\mathcal{Q}_b(\cdot)$ denotes the b-bit quantization function, which processes the real and imaginary components separately [5]. This accounts for a real-valued $N \times K$ system, with $N = 2N_r$ and $K = 2N_t$

$$\mathbf{y} = \mathcal{Q}_b(\mathbf{H}\mathbf{x} + \mathbf{n}), \tag{2}$$

where

$$\mathbf{H}=\begin{bmatrix}\Re\{\bar{\mathbf{H}}\} & -\Im\{\bar{\mathbf{H}}\}\\ \Im\{\bar{\mathbf{H}}\} & \Re\{\bar{\mathbf{H}}\}\end{bmatrix},\quad \mathbf{x}=\begin{bmatrix}\Re\{\bar{\mathbf{x}}\}\\ \Im\{\bar{\mathbf{x}}\}\end{bmatrix},$$
$$\mathbf{y}=\begin{bmatrix}\Re\{\bar{\mathbf{y}}\}\\ \Im\{\bar{\mathbf{y}}\}\end{bmatrix},\quad \mathbf{n}=\begin{bmatrix}\Re\{\bar{\mathbf{n}}\}\\ \Im\{\bar{\mathbf{n}}\}\end{bmatrix}. \tag{3}$$

With respect to the system in (2), the ML detection can be formulated as [3]

$$\min_{\mathbf{x}\in\mathcal{X}^K} f(\mathbf{x})=\sum_{i=1}^{N}-\log\left[\sigma\left(a_i^{\text{up}}\right)-\sigma\left(a_i^{\text{low}}\right)\right], \tag{4}$$

where $a_i^{\text{up}}=q_i^{\text{up}}-\mathbf{h}_i^T\mathbf{x}$ and $a_i^{\text{low}}=q_i^{\text{low}}-\mathbf{h}_i^T\mathbf{x}$, with q_i^{up} and q_i^{low} denoting the upper and lower quantization thresholds to y_i, respectively. $\sigma(\cdot)$ denotes the Sigmoid function and $\mathbf{h}_i^T$ is the i-th row of $\mathbf{H}$. Here, $\mathbf{x}$ belongs to the M-amplitude-shift keying (ASK) constellation set $\mathcal{X}^K$, with $\mathcal{X}=\{\pm1,\pm3,\ldots,\pm(M-1)\}$.

B. Problem Transformation by Exact Penalty Model

The ML problem in (4) is NP-hard, and the complexity of a brute-force search grows exponentially with K. To avoid such exhaustive search, an exact penalty model has been proposed in [9] to relax the ML problem into a continuous form

$$\min_{\mathbf{x}\in\bar{\mathcal{X}}^K} f(\mathbf{x})+\gamma\phi(\mathbf{x}), \tag{5}$$

where $\gamma\geq0$ and $\phi(\mathbf{x})=\sum_{i=1}^{K}\phi(x_i)$,

$$\phi(x)=\min_{j\in\mathcal{U}}\left\{-\frac{1}{2}(x-j)^2+\mathbb{I}_{[-1,1]}(x-j)\right\}. \tag{6}$$

Here, $\mathbb{I}_{[-1,1]}$ denotes the indicator function of $[-1,1]$, $\mathcal{U}=\{0,\pm2,\ldots,\pm(M-2)\}$, and $\bar{\mathcal{X}}=[-(M-1),(M-1)]$. This relaxation enables the use of continuous optimization methods for signal detection. Moreover, it has been shown that, for large enough γ, (5) is an equivalent formulation of (4).

Lemma 1 ([9]). *If $\gamma\geq L_f$, where L_f denotes the Lipschitz constant of ∇f, then any optimal solution to* (5) *is also optimal for* (4).

However, [9] focuses on 1-bit MIMO detection and derives L_f only for that setting. To extend the analysis to multi-bit quantization, L_f needs to be explicitly characterized under the multi-bit model. Moreover, $\phi(\mathbf{x})$ is non-smooth and its gradient is not well-defined everywhere on $\bar{\mathcal{X}}^K$, which complicates the optimization and training process.

C. Decentralized Baseband Processing

To reduce data bandwidth and distribute the complexity in massive MIMO systems, DBP partitions the BS antennas into $C\geq1$ individual DUs, each with $B=N_r/C$ antennas, as illustrated in Fig. 1. Accordingly, the received vector, channel matrix, and quantized thresholds are divided row-wise as $\mathbf{y}=\left[\mathbf{y}_1^T,\ldots,\mathbf{y}_C^T\right]^T$, $\mathbf{H}=\left[\mathbf{H}_1^T,\ldots,\mathbf{H}_C^T\right]^T$, $\mathbf{q}^{\text{up}}=\left[(\mathbf{q}_1^{\text{up}})^T,\ldots,(\mathbf{q}_C^{\text{up}})^T\right]^T$ and $\mathbf{q}^{\text{low}}=\left[(\mathbf{q}_1^{\text{low}})^T,\ldots,(\mathbf{q}_C^{\text{low}})^T\right]^T$, respectively, where $\mathbf{y}_c,\mathbf{q}_c^{\text{up}},\mathbf{q}_c^{\text{low}}\in\mathbb{R}^Q$, $\mathbf{H}_c\in\mathbb{R}^{Q\times K}$ and $Q=2B$. Under DBP architecture, each DU c observes

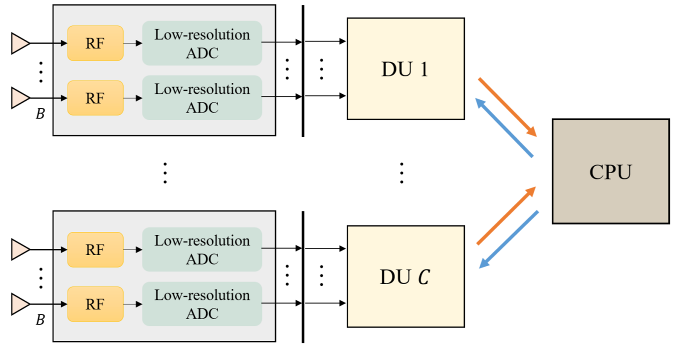


Fig. 1. Illustration of the DBP architecture with C DUs.

$$\mathbf{y}_c=\mathcal{Q}_b(\mathbf{H}_c\mathbf{x}+\mathbf{n}_c), \tag{7}$$

and computes a local estimate to minimize the local objective $f_c(\mathbf{x})=\sum_{i=Q(c-1)+1}^{Qc}-\log\left[\sigma\left(a_i^{\text{up}}\right)-\sigma\left(a_i^{\text{low}}\right)\right]$. These local estimates are aggregated at the CPU to form a global consensus, which is then broadcast back to DUs for subsequent updates, ultimately producing the final estimate.

III. The Proposed Penalty Detection Scheme

In this section, we first derive an explicit exact-penalty condition under multi-bit quantization. For efficient optimization, we then introduce an L^2-optimal smooth penalty to reformulate the ML detection problem. Finally, we detail the distributed HPD algorithm to solve the resulting penalized problem.

A. Exact Penalty Model for Multi-bit Quantization

In [9], an explicit Lipschitz constant L_f is derived for 1-bit MIMO detection. To extend the result to multi-bit quantization, we first characterize the exact L_f under the general quantization model, and then derive a sufficient condition on γ ensuring the equivalence between the ML problem and its penalized formulation.

Theorem 1. *For multi-bit quantization, the ML problem in* (4) *is equivalent to the continuous penalized problem in* (5) *provided that*

$$\gamma\geq L_f, \tag{8}$$

where the Lipschitz constant is given by

$$L_f=\frac{1}{2}\lambda_{\max}\left(\mathbf{H}^T\mathbf{H}\right). \tag{9}$$

Proof. The gradient and Hessian of $f(\mathbf{x})$ are computed as

$$\nabla f(\mathbf{x})=-\mathbf{H}^T\left[\mathbf{1}-\sigma(\mathbf{u}^{\text{up}})-\sigma(\mathbf{u}^{\text{low}})\right], \tag{10}$$

$$\mathcal{H}(\mathbf{x})=\mathbf{H}^T\text{diag}\left(\mathbf{v}^{\text{up}}\right)\mathbf{H}+\mathbf{H}^T\text{diag}\left(\mathbf{v}^{\text{low}}\right)\mathbf{H}, \tag{11}$$

where $\mathbf{u}^{\text{up}}=\mathbf{H}\mathbf{x}-\mathbf{q}^{\text{up}}$, $\mathbf{u}^{\text{low}}=\mathbf{H}\mathbf{x}-\mathbf{q}^{\text{low}}$, $\mathbf{v}^{\text{up}}=\sigma(\mathbf{u}^{\text{up}})\odot\left[\mathbf{1}-\sigma(\mathbf{u}^{\text{up}})\right]$ and $\mathbf{v}^{\text{low}}=\sigma(\mathbf{u}^{\text{low}})\odot\left[\mathbf{1}-\sigma(\mathbf{u}^{\text{low}})\right]$. Based on

(11), for any non-zero vector $\mathbf{w} \in \mathbb{R}^K$, the quadratic form of the Hessian satisfies

$$
\begin{aligned}
\mathbf{w}^T \mathcal{H}(\mathbf{x}) \mathbf{w} &= \mathbf{w}^T \mathbf{H}^T \left[\operatorname{diag}\left(\mathbf{v}^{\text{up}}\right) + \operatorname{diag}\left(\mathbf{v}^{\text{low}}\right)\right] \mathbf{H}\mathbf{w} \\
&= \left\| \sqrt{\operatorname{diag}\left(\mathbf{v}^{\text{up}}\right) + \operatorname{diag}\left(\mathbf{v}^{\text{low}}\right)} \mathbf{H}\mathbf{w} \right\|_2^2 \\
&\leq \frac{1}{2} \|\mathbf{H}\mathbf{w}\|_2^2 \\
&\leq \frac{1}{2} \lambda_{\max}(\mathbf{H}^T\mathbf{H}) \|\mathbf{w}\|_2^2, \qquad (12)
\end{aligned}
$$

where we have used the fact that $\sigma(\nu)\left[1 - \sigma(\nu)\right] \in \left[0, \frac{1}{4}\right]$ for all $\nu \in \mathbb{R}$ [10]. This result implies that $\mathcal{H}(\mathbf{x})$ is upper bounded by $\frac{1}{2}\lambda_{\max}(\mathbf{H}^T\mathbf{H})\mathbf{I}$ in the sense of positive semi-define matrices, which yields a Lipschitz constant of $\nabla f(\mathbf{x})$ under general multi-bit quantization, i.e., $L_f = \frac{1}{2}\lambda_{\max}(\mathbf{H}^T\mathbf{H})$. By Lemma 1, if $\gamma \geq L_f$, then any optimal solution of (5) is also optimal for (4), thus completing the proof. $\square$

Note that, evaluating $\lambda_{\max}(\mathbf{H}^T\mathbf{H})$ exactly requires an eigenvalue decomposition, which is computationally expensive for large-scale systems. To avoid this cost, we exploit the channel hardening property [11] and adopt the following low-complexity choice

$$
\gamma \geq \frac{1}{2}\lambda_{\max}(\mathbf{H}^T\mathbf{H}) \approx \frac{N}{4}\left(1 + \sqrt{\frac{K}{N}}\right)^2, \qquad (13)
$$

which serves as an accurate surrogate when N and K are large under uncorrelated Rayleigh flat-fading channels. With this choice of γ, the ML detection in (4) can be equivalently transformed into the penalized formulation in (5), yielding a continuous detection model. However, the penalty $\phi(\mathbf{x})$ is non-smooth on $\bar{\mathcal{X}}^K$, which limits the use of efficient optimization algorithms and complicates learning-based implementations.

B. Smooth Approximation of $\phi(\mathbf{x})$

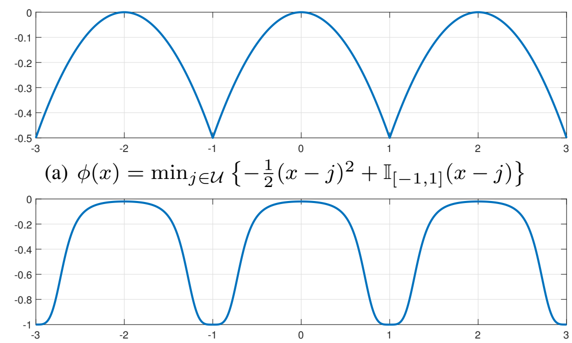
To improve computational efficiency, we introduce a smooth penalty function to promote the discreteness of $\mathbf{x}$:

$$
\psi(\mathbf{x}) = -\sum_{i=1}^{K} \sum_{j \in \mathcal{X}} \frac{1}{100(x_i - j)^4 + 1}, \qquad (14)
$$

and approximate $\phi(\mathbf{x})$ by its smooth surrogate $\alpha\psi(\mathbf{x})$ with the scaling factor $\alpha \geq 0$. For clarity, Fig. 2 illustrates the behavior of penalty functions $\phi(\mathbf{x})$ and $\psi(\mathbf{x})$ under 16-QAM with $\mathcal{X} = \{\pm 1, \pm 3\}$. Notably, both penalties attain their local minima at the constellation points, thereby encouraging $\mathbf{x}$ to approach $\mathcal{X}^K$ during minimization. Whereas $\phi(\mathbf{x})$ is non-smooth and not differentiable at these points, $\psi(\mathbf{x})$ is a smooth surrogate and thus more suitable for efficient optimization.

To select α, we quantify the approximation mismatch using the L^2-norm over $\bar{\mathcal{X}}$. By exploiting the separable structure across entries, we define the per-component approximation error as

$$
\mathcal{J}(\alpha) \triangleq \int_{\bar{\mathcal{X}}} \left[\phi(x) - \alpha\psi(x)\right]^2 \mathrm{d}x, \qquad (15)
$$



(b) $\psi(x) = -\sum_{j \in \mathcal{X}} \frac{1}{100(x-j)^4+1}$

Fig. 2. Illustration of penalty functions component-wise with $\mathcal{X} = \{\pm 1, \pm 3\}$.

which is a strictly convex quadratic function of α. Differentiating $\mathcal{J}(\alpha)$ with respect to α and setting the result to zero yields the unique solution

$$
\alpha^\star = \frac{\int_{\bar{\mathcal{X}}} \phi(x)\psi(x)\mathrm{d}x}{\int_{\bar{\mathcal{X}}} \psi^2(x)\mathrm{d}x}, \qquad (16)
$$

which minimizes $\mathcal{J}(\alpha)$ in (15). For the surrogate penalty in (14), evaluating (16) for square M^2-QAM yields $\alpha^\star \approx 0.428$, with negligible variation across M.

With customized penalty $\psi(\mathbf{x})$ in (14) and $\alpha = \alpha^\star$ in (16), the uplink MIMO detection problem can be reformulated as

$$
\min_{\mathbf{x} \in \bar{\mathcal{X}}^K} \; f(\mathbf{x}) + \gamma\alpha\psi(\mathbf{x}), \qquad (17)
$$

which will be efficiently solved by the proposed accelerated gradient-based method in the next subsection.

C. The Proposed Distributed HPD Algorithm

The penalty term in (17) makes the problem non-convex, and directly using a large penalty may lead to poor local minima [13]. We therefore adopt a homotopy strategy that gradually increases the penalty parameter from a small γ^0 to large values, and traces the solution path of (17) toward the ML solution. At each iteration, the penalty parameter is updated as

$$
\gamma^t = \gamma^{t-1}\varrho, \qquad (18)
$$

where $\varrho > 1$ controls the growth rate and t denotes the iteration index. Under this architecture, the proposed HPD algorithm solves the resulting penalized problem in a distributed manner. Specifically, HPD generates the local estimates $\mathbf{x}_c^t$ at each DU c by

$$
\mathbf{x}_c^t = \Pi_{\bar{\mathcal{X}}^K}\left\{\mathbf{z}^{t-1} - \mu\left[\nabla f_c(\mathbf{z}^{t-1}) + \frac{\alpha\gamma^t}{C}\nabla\psi(\mathbf{z}^{t-1})\right]\right\}, \qquad (19)
$$

with the local gradient

$$
\nabla f_c(\mathbf{z}^{t-1}) = -\mathbf{H}_c^T\left[\mathbf{1} - \sigma(\mathbf{H}_c\mathbf{z}^{t-1} - \mathbf{q}_c^{\text{up}}) - \sigma(\mathbf{H}_c\mathbf{z}^{t-1} - \mathbf{q}_c^{\text{low}})\right], \qquad (20)
$$

$$
\nabla\psi(\mathbf{z}^{t-1}) = \sum_{j \in \mathcal{X}} \frac{400(\mathbf{z}^{t-1} - j\mathbf{1})^3}{(100(\mathbf{z}^{t-1} - j\mathbf{1})^4 + \mathbf{1})^2}. \qquad (21)
$$

Algorithm 1: Proposed Distributed HPD Detector

Input : $\mathbf{y}_c$, $\mathbf{H}_c$, $\mathbf{q}_c^{\text{up}}$, $\mathbf{q}_c^{\text{low}}$, μ, γ^0, ϱ, L

```
Initialization: z^0 = s^0 = 0, t = 0, α = ∫_X̄ φ(x)ψ(x)dx / ∫_X̄ ψ²(x)dx
repeat
    t ← t + 1
    γ^t = γ^{t-1} ϱ
    for c = 1, 2, ..., C do                         // in DUs
        evaluate ∇f_c(z^{t-1}) and ∇ψ(z^{t-1}) by (20)-(21)
        update x_c^t by (19)
    end
    s^t = (1/C) Σ_{c=1}^{C} x_c^t                    // in CPU
    z^t = s^t + t/(t+3) (s^t − s^{t-1})
until γ^t ≥ (N/4)(1 + sqrt(K/N))^2 or t = L;
```

Output: estimated transmit signal: $\widehat{\mathbf{x}} = \lceil \mathbf{s}^t \rfloor_{\mathcal{Q}} \in \mathcal{X}^K$

Here, the power and division operations are performed element-wise. $\Pi_{\bar{\mathcal{X}}^K}(\cdot)$ projects each component onto $\bar{\mathcal{X}}$, and $\mu > 0$ denotes the step size. The local estimates $\mathbf{x}_c^t$ at all DUs are then uploaded to the CPU, where they are aggregated to form the global estimate

$$\mathbf{s}^t = \frac{1}{C}\sum_{c=1}^{C} \mathbf{x}_c^t, \tag{22}$$

which is followed by a momentum update

$$\mathbf{z}^t = \mathbf{s}^t + \frac{t}{t+3}\left(\mathbf{s}^t - \mathbf{s}^{t-1}\right). \tag{23}$$

The consensus vector $\mathbf{z}^t$ is then fed back to each DU for the next iteration. For initialization, we set $\mathbf{z}^0 = \mathbf{s}^0 = \mathbf{0}$ and choose α according to (16). The iterations terminate once γ^t exceeds the threshold in (13) or the iteration index t reaches the maximum number L. At that point, CPU rounds the global estimate $\mathbf{s}^t$ to the nearest constellation points for final detection output. The overall procedure of the proposed distributed HPD detector is summarized in Algorithm 1.

IV. The Proposed Distributed FPD-Net

In this section, we first develop a multi-term smoothing framework for $\phi(\mathbf{x})$ and propose a fusion penalty model with reduced approximation error. We then employ a deep unfolding network to improve the detection performance.

A. Multi-Term Smoothing Framework of $\phi(\mathbf{x})$

To improve the approximation quality of the non-smooth penalty $\phi(\mathbf{x})$, we construct a general multi-term smoothing framework. Specifically, we consider a family of smooth functions $\{g_k(\mathbf{x})\}_{k\geq 1}$ and form their finite linear combinations as

$$\mathcal{G}_n = \left\{ \sum_{k=1}^{n} \alpha_k g_k(\mathbf{x}) : \alpha_k \in \mathbb{R} \right\}. \tag{24}$$

Here, $\{\alpha_k\}_{k=1}^n$ are the combination coefficients and n is the number of terms. Then, we approximate $\phi(\mathbf{x})$ by a smooth function $G(\mathbf{x}) \in \mathcal{G}_n$, and define the worst-case approximation error as

$$\varepsilon_n \triangleq \inf_{G(\mathbf{x})\in\mathcal{G}_n} \left\|\phi(\mathbf{x}) - G(\mathbf{x})\right\|_\infty, \tag{25}$$

where $\|\cdot\|_\infty$ denotes the uniform norm on $\bar{\mathcal{X}}^K$, i.e., $\|\Psi\|_\infty = \sup_{\mathbf{x}\in\bar{\mathcal{X}}^K} |\Psi(\mathbf{x})|$. It is well known that if the linear span of $\{g_k(\mathbf{x})\}_{k\geq 1}$ is dense in $\bar{\mathcal{X}}^K$ under the uniform norm, then ε_n can be made arbitrarily small as n increases [12], which leads to a default configuration that $\varepsilon_n \to 0$ as $n \to \infty$.

The following Lemma shows that the approximation accuracy of $G(\mathbf{x})$ improves monotonically as n grows, as reflected by the non-increasing error sequence $\{\varepsilon_n\}$.

Lemma 2. *For all $n \geq 1$, the approximation error ε_n satisfies*

$$\varepsilon_{n+1} \leq \varepsilon_n. \tag{26}$$

Proof. By definition, $\mathcal{G}_n \subset \mathcal{G}_{n+1}$. Thus, we obtain

$$\inf_{G(\mathbf{x})\in\mathcal{G}_{n+1}} \left\|\phi(\mathbf{x}) - G(\mathbf{x})\right\|_\infty \leq \inf_{G(\mathbf{x})\in\mathcal{G}_n} \left\|\phi(\mathbf{x}) - G(\mathbf{x})\right\|_\infty, \tag{27}$$

which completes the proof. □

Let $f^* = \min_{\mathbf{x}\in\mathcal{X}^K} f(\mathbf{x})$ denote the optimal ML objective value. For each n, choose $G_n(\mathbf{x}) \in \mathcal{G}_n$ such that

$$\left\|\phi(\mathbf{x}) - G_n(\mathbf{x})\right\|_\infty \leq \varepsilon_n + \frac{1}{n}, \tag{28}$$

and define the corresponding smooth surrogate optimal value

$$V_n = \min_{\mathbf{x}\in\bar{\mathcal{X}}^K} \left[f(\mathbf{x}) + \bar{\gamma} G_n(\mathbf{x})\right], \tag{29}$$

with a penalty parameter $\bar{\gamma} \geq \frac{1}{2}\lambda_{\max}\big(\mathbf{H}^T\mathbf{H}\big)$. Then, as n tends to infinity, the following Lemma demonstrates that the ML performance can be asymptotically achieved.

Lemma 3. *As $n \to \infty$, the following convergence holds*

$$\lim_{n\to\infty} |V_n - f^*| = 0. \tag{30}$$

Proof. By the exact penalty property in Theorem 1, we have

$$f^* = \min_{\mathbf{x}\in\mathcal{X}^K} f(\mathbf{x}) = \min_{\mathbf{x}\in\bar{\mathcal{X}}^K} \left[f(\mathbf{x}) + \bar{\gamma}\phi(\mathbf{x})\right]. \tag{31}$$

For all $n \geq 1$, it then follows that

$$\begin{aligned}
|V_n - f^*| &= \Big| \min_{\mathbf{x}\in\bar{\mathcal{X}}^K} \big[f(\mathbf{x}) + \bar{\gamma} G_n(\mathbf{x})\big] - \min_{\mathbf{x}\in\bar{\mathcal{X}}^K} \big[f(\mathbf{x}) + \bar{\gamma}\phi(\mathbf{x})\big] \Big| \\
&\leq \Big\| \big[f(\mathbf{x}) + \bar{\gamma} G_n(\mathbf{x})\big] - \big[f(\mathbf{x}) + \bar{\gamma}\phi(\mathbf{x})\big] \Big\|_\infty \\
&= \bar{\gamma} \left\|G_n(\mathbf{x}) - \phi(\mathbf{x})\right\|_\infty \\
&\leq \bar{\gamma}\left(\varepsilon_n + \tfrac{1}{n}\right).
\end{aligned} \tag{32}$$

Since $\varepsilon_n \to 0$ as $n \to \infty$, the right-hand side of (32) converges to zero, completing the proof. □

Motivated by Lemma 2, we extend the single-penalty formulation in (17) to a fusion model that combines three penalties, thereby reducing the approximation error to $\phi(\mathbf{x})$. This leads to the following detection problem

$$\min_{\mathbf{x}\in\bar{\mathcal{X}}^K} \ f(\mathbf{x}) + \gamma \sum_{i=1}^{3} \alpha_i g_i(\mathbf{x}), \tag{33}$$

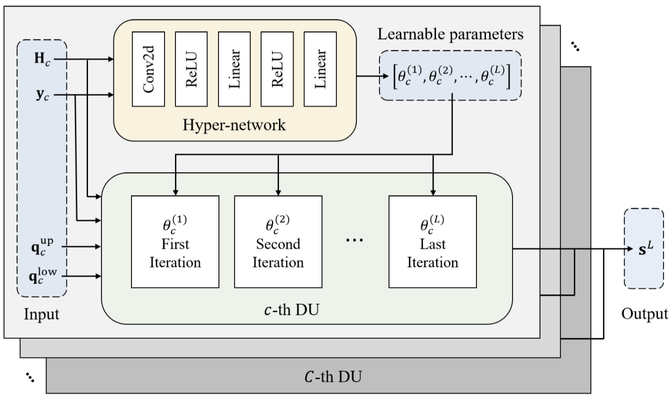


Fig. 3. The structure of the FPD-Net detector.

with

$$g_1(\mathbf{x}) = -\sum_{i=1}^{K}\sum_{j\in\mathcal{X}} \frac{1}{\eta(x_i - j)^4 + 1}, \tag{34}$$

$$g_2(\mathbf{x}) = -\sum_{i=1}^{K}\sum_{j\in\mathcal{X}} e^{-\beta(x_i-j)^2}, \tag{35}$$

$$g_3(\mathbf{x}) = \sum_{i=1}^{K} \log\left[\cos(\pi x_i) + \varphi\right]. \tag{36}$$

Here, $\alpha_i \geq 0$, $i \in \{1, 2, 3\}$, are the penalty weights controlling the influence of the corresponding penalty terms, and $\eta, \beta > 0$, $\varphi > 1$ serve as shape parameters.

Each individual penalty is designed to attain local minima at the constellation points. Therefore, the fused penalty retains a strong attraction toward the discrete set $\mathcal{X}^K$. Moreover, by Lemma 2, the use of multiple penalty terms can improve the approximation of $\phi(\mathbf{x})$ with suitable $\{\alpha_i\}_{i=1}^3$. However, finding jointly optimal weights under the uniform-norm criterion is analytically intractable. Therefore, we let these penalty weights be learned from data through the subsequent deep learning (DL) framework.

B. The Proposed Distributed FPD-Net Detector

To effectively solve the fusion detection problem in (33), we upgrade the HPD algorithm by incorporating the additional penalties and embedding it into a DNN, where the parameters are learned via data-driven training. The gradients ∇f_c and $\nabla\psi$ in HPD are both computed at the common estimate $\mathbf{z}^{t-1}$, and this parallel update underuses intermediate information and may slow down convergence. To address this issue, fusion penalty detection network (FPD-Net) modifies the updates into a sequential form and introduces three auxiliary variables $\mathbf{b}_c^t$, $\mathbf{p}_c^t$ and $\mathbf{d}_c^t$, which are updated successively. Specifically, at the c-th DU, the data detection is performed as follows

$$\mathbf{b}_c^t = \mathbf{z}^{t-1} + \mu_1^t \mathbf{H}_c^T\left[\mathbf{1} - \sigma(\mathbf{H}_c\mathbf{z}^{t-1} - \mathbf{q}_c^{\text{up}}) - \sigma(\mathbf{H}_c\mathbf{z}^{t-1} - \mathbf{q}_c^{\text{low}})\right], \tag{37}$$

$$\mathbf{p}_c^t = \mathbf{b}_c^t - \mu_2^t \sum_{j\in\mathcal{X}} \frac{4\eta^t(\mathbf{b}_c^t - j\mathbf{1})^3}{(\eta^t(\mathbf{b}_c^t - j\mathbf{1})^4 + \mathbf{1})^2}, \tag{38}$$

Algorithm 2: Proposed Distributed FPD-Net Detector

Input : $\mathbf{y}_c$, $\mathbf{H}_c$, $\mathbf{q}_c^{\text{up}}$, $\mathbf{q}_c^{\text{low}}$, θ_c^t, L

Initialization: $\mathbf{z}^0 = \mathbf{s}^0 = \mathbf{0}$

for $t = 1, 2, \cdots, L$ **do**

  for $c = 1, 2, \cdots, C$ **do** `// in DUs`

    update $\mathbf{b}_c^t$, $\mathbf{p}_c^t$, $\mathbf{d}_c^t$ and $\mathbf{x}_c^t$ using (37)-(40)

  end

  $\mathbf{s}^t = \frac{1}{C}\sum_{c=1}^{C}\mathbf{x}_c^t$ `// in CPU`

  $\mathbf{z}^t = \mathbf{s}^t + \frac{t}{t+3}\left(\mathbf{s}^t - \mathbf{s}^{t-1}\right)$

end

Output: estimated transmit signal: $\widehat{\mathbf{x}} = \lceil \mathbf{s}^L \rfloor_{\mathcal{Q}} \in \mathcal{X}^K$

$$\mathbf{d}_c^t = \mathbf{p}_c^t - \mu_3^t \sum_{j\in\mathcal{X}} 2\beta^t(\mathbf{p}_c^t - j\mathbf{1}) \odot e^{-\beta^t(\mathbf{p}_c^t - j\mathbf{1})^2}, \tag{39}$$

$$\mathbf{x}_c^t = \Pi_{\bar{\mathcal{X}}^K}\left\{\mathbf{d}_c^t + \mu_4^t \frac{\pi\sin(\pi\mathbf{d}_c^t)}{\cos(\pi\mathbf{d}_c^t) + \varphi^t\mathbf{1}}\right\}. \tag{40}$$

The penalty parameters are omitted here, since the layer-dependent step sizes take over the same role.

Moreover, for further adaptability and detection performance of FPD-Net, we introduce a hyper-network to learn parameters. Given the structure in Fig. 3, the hyper-network takes $(\mathbf{H}_c, \mathbf{y}_c)$ as inputs and first extracts features with convolutional layers. These features are then combined and passed through a two-layer fully connected network with ReLU and linear activation, which outputs the layer-dependent learnable parameters $\theta_c^t = \{\mu_1^t, \mu_2^t, \mu_3^t, \mu_4^t, \eta^t, \beta^t, \varphi^t\}$. In the FPD-Net, the learnable parameters are trained by minimizing the mean-squared error (MSE) loss function

$$\mathcal{L}_{\text{mse}} = \mathrm{E}\left[\|\mathbf{s}^L - \mathbf{x}\|_2^2\right], \tag{41}$$

where $\mathbf{s}^L$ denotes the output of FPD-Net and $\mathbf{x}$ is the training label. For a better understanding, the proposed FPD-Net is summarized in Algorithm 2.

V. Numerical Results

In our simulation, the channel elements are modeled as i.i.d. $\mathcal{CN}(0, 1)$ random variables. We set the numbers of receive antennas, transmit antennas, and DUs to 64, 8, and 4, respectively. All simulations use the non-uniform quantizers in [14]. As baselines, we include the centralized BMMSE detector [2] and the decentralized CG and ADMM detectors [7] for comparison. The distributed HPD algorithm is implemented with $\mu = 0.2$, $\gamma^0 = 1$ and $\varrho = 1.5$. For FPD-Net, the training is carried out offline, and the trained parameters are then used during the online detection phase. We implement FPD-Net using the PyTorch DL library and train it with the Adam optimizer [15], using a batch size of 1000 and a learning rate of 0.001. For all iterative algorithms, the maximum number of iterations is set to $L = 5$.

Fig. 4 compares the detection performance of the proposed HPD and FPD-Net detectors with several existing detection schemes for 4-QAM and 1-bit ADCs. Since CG and ADMM are quantization-unaware, they exhibit pronounced error floors

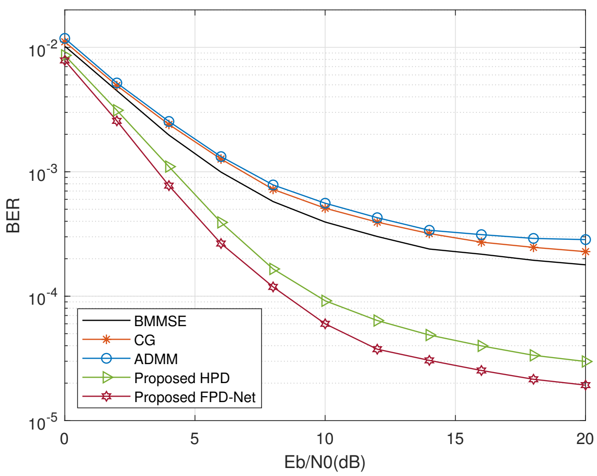


Fig. 4. BER performance comparison between proposed HPD and FPD-Net detectors and existing methods with 4-QAM modulation and 1-bit ADCs.

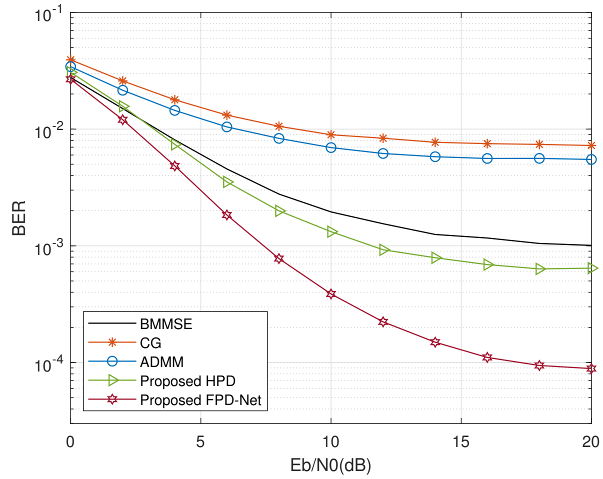


Fig. 5. BER performance comparison between proposed HPD and FPD-Net detectors and existing methods with 16-QAM modulation and 2-bit ADCs.

under such severe quantization distortion. Notably, the proposed HPD detector outperforms the conventional BMMSE algorithm by achieving lower BER. Furthermore, with the aid of DL, FPD-Net provides an additional performance gain over HPD and achieves the best detection accuracy.

Fig. 5 extends the BER performance to system with 16-QAM modulation and 2-bit quantization. Although BMMSE, CG, and ADMM benefit from higher quantization resolution, their BER performance in distributed massive MIMO systems remains unsatisfactory. Similar to the observations in Fig. 4, both HPD and FPD-Net outperform BMMSE while maintaining lower computational complexity and fronthaul bandwidth requirements. These results highlight the scalability and efficiency of the proposed detectors, demonstrating their potential as practical detection solutions for distributed massive MIMO systems with low-resolution ADCs.

VI. Conclusion

In this paper, we proposed two efficient detectors for quantized and distributed massive MIMO systems: the HPD and FPD-Net. By introducing a tailored penalty function into the ML detection framework, HPD effectively solves the resulting problem and achieves notable BER gains with low complexity and bandwidth costs. Furthermore, FPD-Net solves a fusion penalized detection problem and learns the step sizes and shape parameters via a hyper-network, thereby further enhancing detection performance and accelerating convergence. Numerical results verify that both methods offer clear advantages over conventional detectors, highlighting the potential of smooth penalty-based designs for practical uplink massive MIMO systems.

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