Reduced-Basis Constrained Tree Search for Large-Scale MIMO Detection

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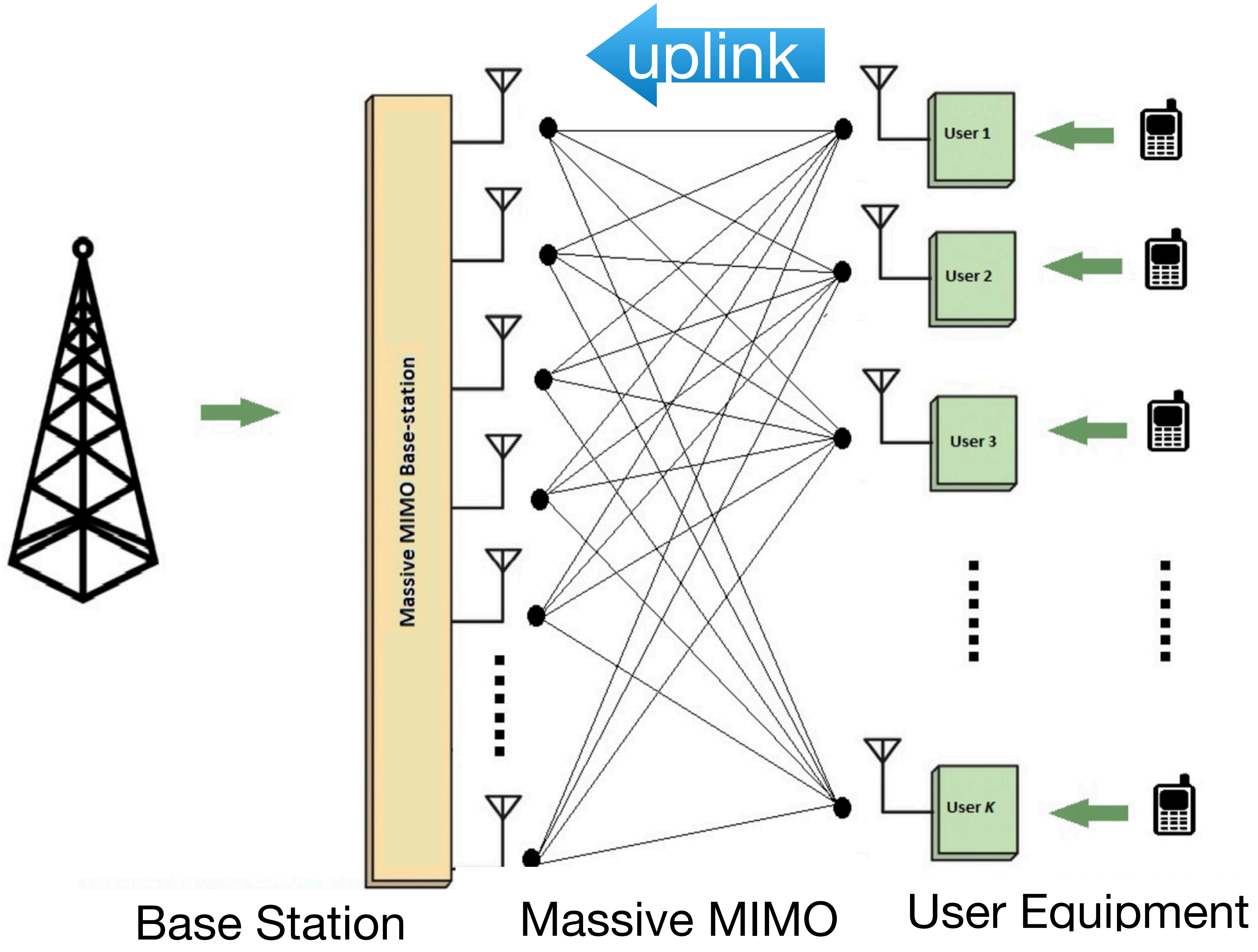
IEEE VTS
VTC2026-Spring

Outlines

- **Introduction**
- **System Model & Lattice Reduction**
- **Reduced-basis Constrained Tree Search**
- **Numerical results**
- **Conclusion**

Introduction

- Large-scale MIMO Detection
  - On the uplink side



- ***Tree-search-based methods***
  - Sphere decoding
  - fixed-complexity variants and other structured tree search schemes
    - k-best, FSD

Can we keep the advantage of tree search, but reduce the number of expanded candidates?

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- **System model**
  - real-valued $n \times n$ large-scale MIMO system
    - $\mathbf{c} = \mathbf{H}\mathbf{x} + \mathbf{w}$
      - $\mathbf{x} \in \mathscr{X}^n$: transmitted signal
      - $\mathbf{c} \in \mathbb{R}^n$: received signal
      - $\mathbf{H} \in \mathbb{R}^{n \times n}$: channel matrix with i.i.d. real-valued Gaussian entries
      - $\mathbf{w} \in \mathbb{R}^n$: AWGN vector with zero mean and variance σ_{w}^2
    - $\mathscr{X} = \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\}$: $\sqrt{M}$-ary PAM constellation $\rightarrow$ M-QAM
  - Optimal ML detection: $\hat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\mathbf{x} \in \mathscr{X}^n} \|\mathbf{H}\mathbf{x} - \mathbf{c}\|^2$

- **Lattice reduction**
  - $\Lambda = \{\mathbf{H}\mathbf{x} : \mathbf{x} \in \mathbb{Z}^n\}$: n-dimensional lattice generated by the full-rank matrix $\mathbf{H}$ (lattice basis)
  - $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{T}$
    - $\mathbf{T} \in \mathbb{Z}^{n \times n}$: unimodular transformation
  - LLL-reduced：
    - $|\mu_{i,j}| \leq \frac{1}{2}, \quad 1 \leq j < i \leq n$
    - $\|\hat{\mathbf{h}}_i + \mu_{i,i-1}\hat{\mathbf{h}}_{i-1}\|^2 \geq \delta \|\hat{\mathbf{h}}_{i-1}\|^2, \quad 1 < i \leq n,$
      - $\hat{\mathbf{h}}_i$: i-th Gram-Schmidt (GS) vector
      - $\mu_{i,j}$: corresponding GS coefficients
      - $\delta \in (1/4, 1)$: Lovász constant
    - $\|\hat{\mathbf{h}}_i\|^2 \geq (\delta - \mu_{i,i-1}^2)\|\hat{\mathbf{h}}_{i-1}\|^2 \geq (\delta - \frac{1}{4})\|\hat{\mathbf{h}}_{i-1}\|^2$
      - **Improved orthogonality**

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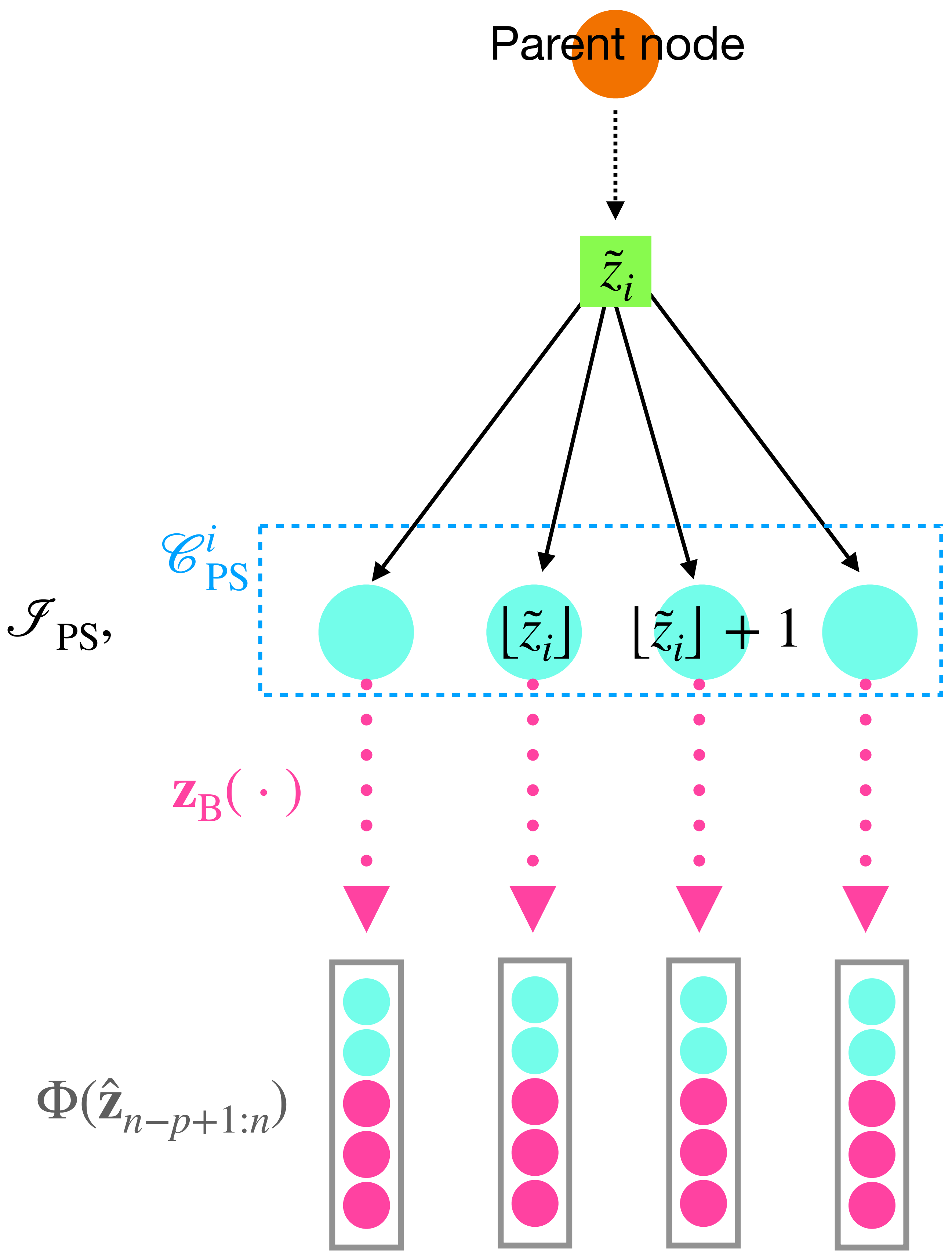
- **Algorithm Description -** (Reduced-basis Constrained Tree Search, RB-CTS)
  - System after preprocessing: $\mathbf{y} = \mathbf{R}\mathbf{z} + \mathbf{n}$
    - Ordering & LLL reduction & QR decomposition
  - Soft center:
    $$\tilde{z}_i = \frac{1}{r_{i,i}}\left(y_i - \sum_{k=i+1}^{n} r_{i,k}\hat{z}_k\right)$$
  - Two phases:
    - Constrained Prefix Search (PS)
      - Index set: $\mathcal{I}_{\mathrm{PS}} = \{n, n-1, \ldots, n-p+1\}$
      - Selecting the $2S$ nearest integer points centered at $\lfloor \tilde{z}_i \rceil$: $\mathcal{C}^i_{\mathrm{PS}} \triangleq \{\lfloor \tilde{z}_i \rceil - S + 1, \ldots, \lfloor \tilde{z}_i \rceil, \ldots, \lfloor \tilde{z}_i \rceil + S\}, \quad i \in \mathcal{I}_{\mathrm{PS}},$
      - Resulting prefix candidate set: $\mathcal{Z}_{\mathrm{PS}} \triangleq \mathcal{C}^{n-p+1}_{\mathrm{PS}} \times \cdots \times \mathcal{C}^{n}_{\mathrm{PS}}$
      - Surviving prefix vector: $\hat{\mathbf{z}}_{n-p+1:n} \in \mathcal{Z}_{\mathrm{PS}}$
    - Babai completion
      - Index set $\mathcal{I}_{\mathrm{B}} = \{n-p, n-p-1, \ldots, 1\}$
      - $\hat{z}_i = \lfloor \tilde{z}_i \rceil, \qquad i \in \mathcal{I}_{\mathrm{B}},$
      - Yielding $\mathbf{z}_{\mathrm{B}}(\hat{\mathbf{z}}_{n-p+1:n}) \triangleq \hat{\mathbf{z}}_{1:n-p}$
  - Final hard decision:
    - Completed lattice point corresponding to a given prefix $\hat{\mathbf{z}}_{n-p+1:n}$ is constructed via the mapping: $\Phi(\hat{\mathbf{z}}_{n-p+1:n}) \triangleq \begin{bmatrix} \mathbf{z}_{\mathrm{B}}(\hat{\mathbf{z}}_{n-p+1:n}) \\ \hat{\mathbf{z}}_{n-p+1:n} \end{bmatrix} \in \mathbb{Z}^n$
    - $\hat{\mathbf{z}}_{\text{RB-CTS}} = \arg\min_{\mathbf{z} \in \Phi(\mathcal{Z}_{\mathrm{PS}})} \left\| \mathbf{y} - \mathbf{R}\mathbf{z} \right\|^2$

For a real-valued system, selecting the PS depth as $p \geq 2\left(\sqrt{n/2} - 1\right)$ is sufficient in practice to retain full receive diversity of the proposed RB-CTS, as will be seen later.



- **Reliability of the PS Phase**

Lemma 1. *The probability of successful prefix retention during the PS phase satisfies*

$$\Pr(\mathbf{x} \in \mathcal{S}_{\mathrm{PS}}) \geq 1 - p \cdot \exp\left(-\frac{S^2 \min_i |r_{i,i}|^2}{2\kappa\sigma_{\mathrm{n}}^2}\right), \tag{20}$$

where p is the PS depth, S is the per-layer prefix search radius, and $\kappa > 0$ is a constant determined by the residual interference level.

  - LLL reduction — two beneficial effects on this bound
    - balancing the diagonal entries $|r_{i,i}| \rightarrow$ increase the minimum diagonal entry $\min_i |r_{i,i}|$
    - exhibits weaker inter-layer coupling $\rightarrow$ reduces the interference-related constant κ
  - a relatively small integer radius S is sufficient to ensure a high probability of successful prefix retention $\rightarrow$ reducing the number of visited search nodes

- **Complexity Analysis**
  - Number of visited tree nodes N_{node}
    - PS phase:
      $$N_{\mathrm{PS}} = \sum_{\ell=1}^{p} (2S)^{\ell} = \frac{(2S)^{p+1} - 2S}{2S - 1}$$
    - Babai completion phase:
      $N_{\mathrm{B}} = (n - p)(2S)^p$
    - In total:
      $$N_{\text{node}} = \frac{(2S)^{p+1} - 2S}{2S - 1} + (n - p)(2S)^p = O\left(n(2S)^p\right)$$
  - Times FLOPs per visited node:
    - $C_{\text{RB-CTS}} = O(n)\, N_{\text{node}} = O\left(n^2(2S)^p\right)$
  - Insensitive to the modulation order for a fixed small integer S

- **Receive Diversity Analysis**

Theorem 1. *For an i.i.d. Rayleigh fading MIMO channel, the proposed RB-CTS achieves the full receive diversity order, i.e.,*

$$\lim_{\rho\to\infty} -\frac{\log P_e^{\text{RB-CTS}}}{\log \rho} = n, \tag{30}$$

where $\rho = 1/\sigma_{\mathrm{n}}^2$ *denotes the signal-to-noise ratio (SNR).*

- **Full** receive diversity order
- The Lovász constant δ influences the SNR gap but not the diversity order.

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Numerical Results

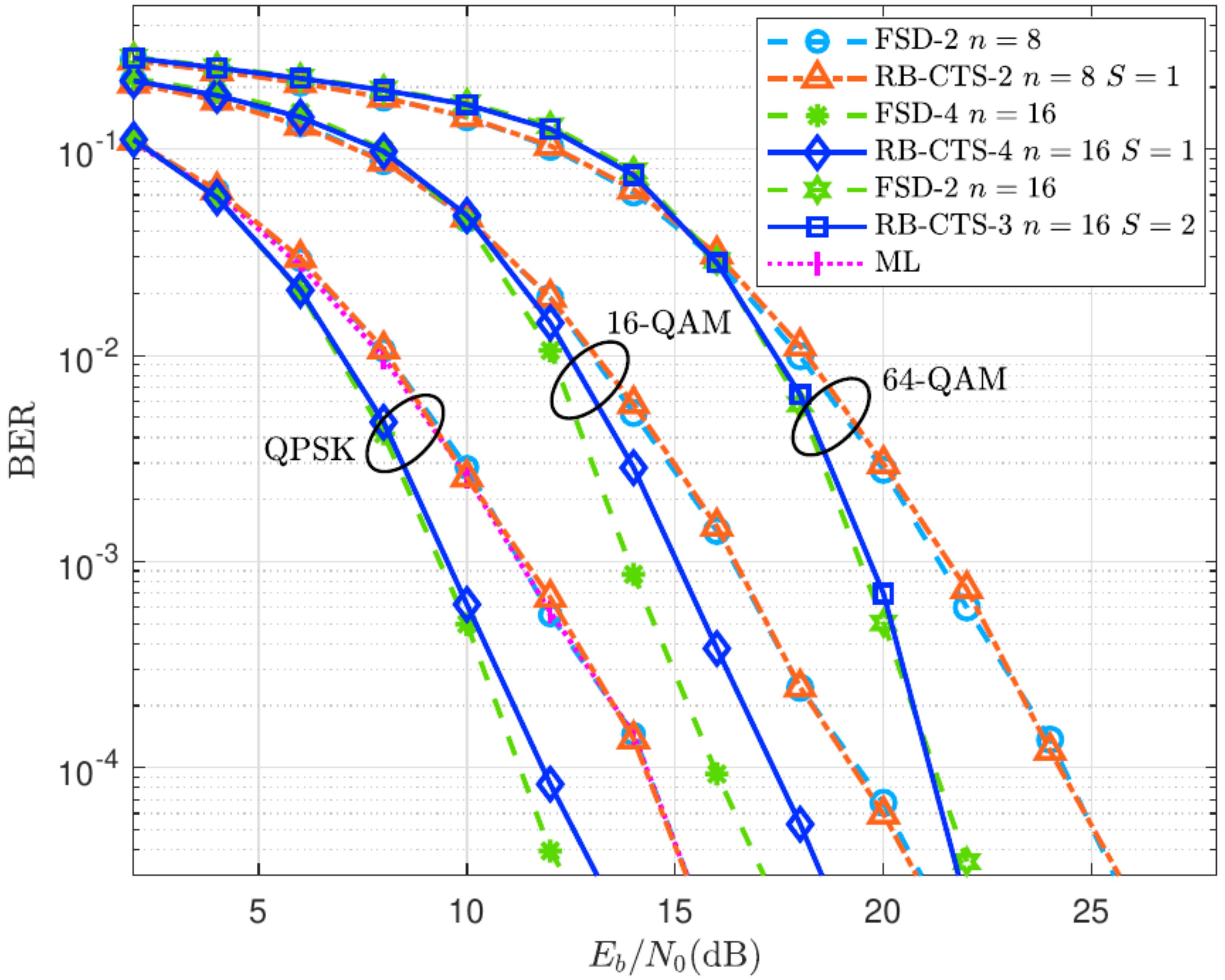

Fig. 1. Performance comparison for $n \times n$ real-valued MIMO systems.

- RB-CTS achieves the expected **full diversity order**

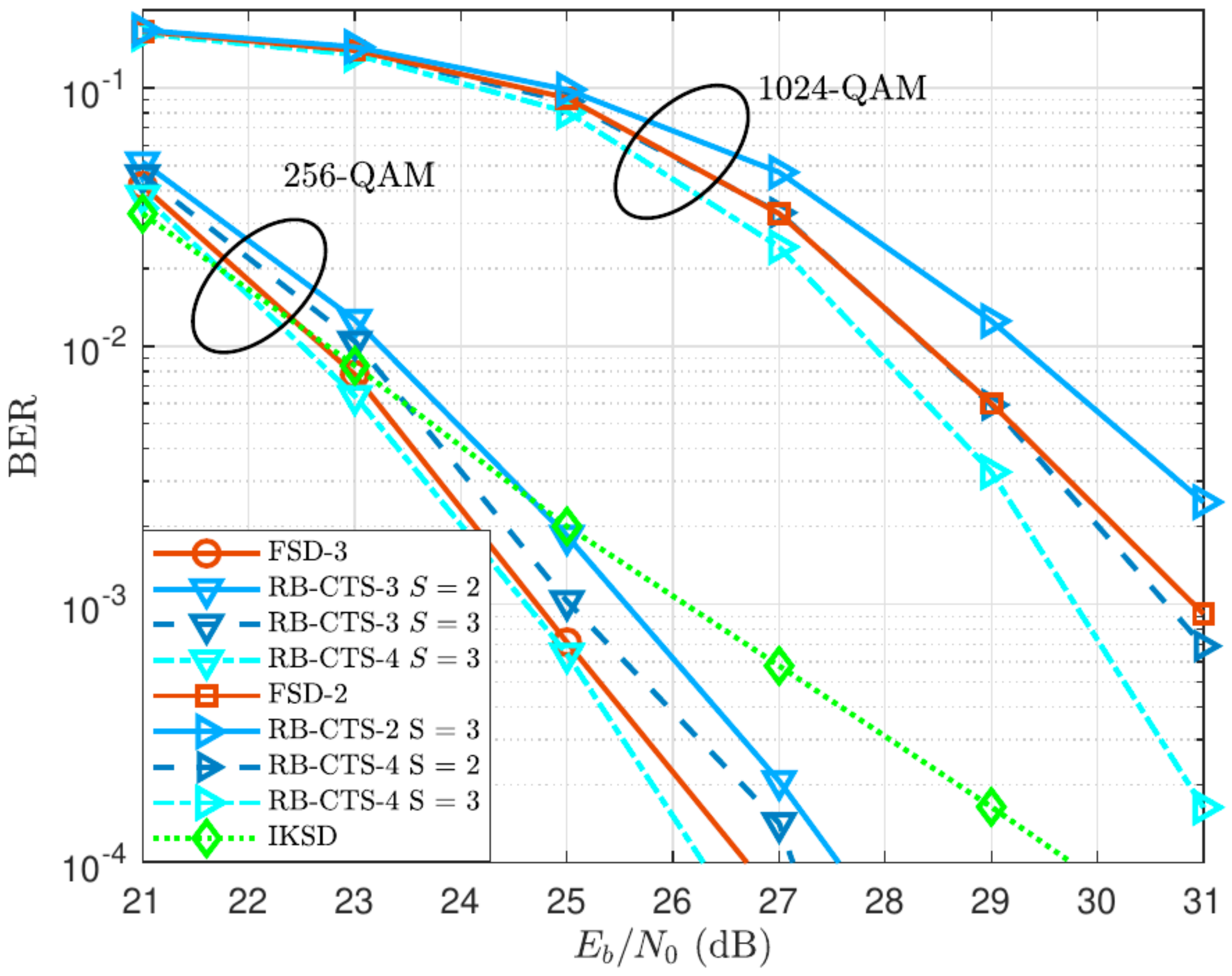

Fig. 2. Performance comparison under 32×32 real-valued MIMO system.

Method	256-QAM	1024-QAM
RB-CTS	1,940 ($p = 3,\ S = 2$)	1,122 ($p = 2,\ S = 3$)
	6,522 ($p = 3,\ S = 3$)	7,508 ($p = 4,\ S = 2$)
	37,842 ($p = 4,\ S = 3$)	37,842 ($p = 4,\ S = 3$)
FSD	123,152	31,776

- Favorable performance-complexity trade-off in high-order modulation scenarios

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- **Conclusion**
- The proposed reduced-basis constrained tree search (**RB-CTS**) framework is distinguished by
    - a modulation-insensitive constrained prefix search with fixed candidate size
        - reliability: prefix-retention probability analysis
        - full receive diversity: despite the reduced search space

Thanks for listening, any questions?

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