

Block-Level Precoding for MIMO-ISAC Based on Interference Exploitation

Chengzhe Weng, Bin Yan, Zheng Wang, Yongming Huang

School of Information Science and Engineering, Southeast University, Nanjing, 210096, China

Email: {chengzhe, bin_yan, zheng_wang, huangym}@seu.edu.cn

Abstract—In this paper, the technique of constructive interference (CI) is introduced to improve the performance of multiple-input multiple-output (MIMO) integrated sensing and communication (ISAC) systems. First, we formulate a non-convex sensing-oriented precoding problem under communication CI constraints, which can be approximately solved via successive convex approximation (SCA). To reduce computational complexity, we further derive the *Lagrange* dual of the SCA sub-problem. By eliminating the explicit second-order cone constraint in the primal sub-problem, the dual formulation enables a more efficient solution procedure, leading to the proposed CI-SCA-Dual algorithm. Simulation results show that the proposed CI-SCA-Dual precoding method achieves significant ISAC performance improvement with limited computational complexity increment.

Index Terms—integrated sensing and communication (ISAC), constructive interference (CI), block-level precoding (BLP).

I. INTRODUCTION

In 6G wireless communication systems, higher frequency bands, wider bandwidths, and larger-scale antenna arrays enable high-precision, high-resolution sensing, thus allowing the integration of sensing and communication (ISAC) in a single system. The combination of multiple-input multiple-output (MIMO) and ISAC, on the one hand, provides extremely high spatial resolution; on the other hand, it assists in channel state information acquisition and signal processing [1]. Since communication and sensing share the same hardware platform, spectrum resources, and transmit power, efficient precoding design is essential for resource allocation.

Traditionally, multiuser interference is regarded as a detrimental factor. However, the dirty paper coding (DPC) method in [2] implies that it is optimal to code along interference. Subsequently, this idea has been further developed under the framework of symbol-level precoding (SLP) based on constructive interference (CI), pushing the received signal away from the detection boundaries, which contributes to increasing the useful signal power [3]. Recently, [4] proposes a CI-based block-level precoding (BLP) method. Compared with CI-SLP, which requires symbol-by-symbol optimization within each block, CI-BLP updates the precoder only once per block, thus greatly reducing the computational cost.

In this paper, we introduce CI-BLP into QAM-modulated MIMO-ISAC to increase the useful signal power, which is

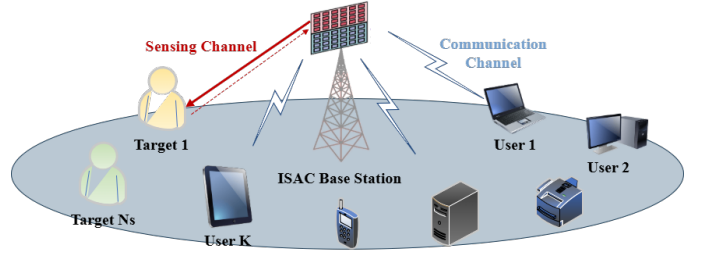


Fig. 1. MIMO-ISAC Downlink System.

more challenging than the QPSK-limited scheme in [5]. However, incorporating CI-BLP into MIMO-ISAC tightly couples CI-constrained communication with Cramér-Rao bound (CRB)-oriented sensing, resulting in a highly non-convex precoding problem. To address this challenge, we employ successive convex approximation (SCA) to obtain a sub-optimal solution. Furthermore, since the SCA sub-problem satisfies *Slater's* condition, we derive its *Lagrange* dual formulation, which enables a more efficient solution procedure.

II. SYSTEM MODEL

A. MIMO-ISAC

We consider a MIMO-ISAC base station (BS) equipped with N_t transmit antennas and N_r receive antennas. It serves K downlink single-antenna users while it senses N_s point targets. Let $\mathbf{S} = [\mathbf{s}^1, \mathbf{s}^2, \dots, \mathbf{s}^{N_s}] \in \mathbb{C}^{K \times N}$ denote the data-symbol matrix, assumed to be drawn from a QAM constellation, where N is the block length. After linear precoding with $\mathbf{W} \in \mathbb{C}^{N_t \times K}$, the transmitted signal matrix is given by:

$$\mathbf{X} = \mathbf{W}\mathbf{S} \in \mathbb{C}^{N_t \times N}. \quad (1)$$

On the communication side, the received signal matrix $\mathbf{Y}_C \in \mathbb{C}^{K \times N}$ at the users is modeled as:

$$\mathbf{Y}_C = \mathbf{H}\mathbf{X} + \mathbf{Z}_C. \quad (2)$$

Here $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K]^T \in \mathbb{C}^{K \times N_t}$ denotes the Rayleigh-fading channel matrix and is assumed known at the BS. In addition, $\mathbf{Z}_C \in \mathbb{C}^{K \times N}$ denotes a complex additive white Gaussian noise (AWGN) matrix with variance σ_C^2 .

On the sensing side, the received signal matrix $\mathbf{Y}_S \in \mathbb{C}^{N_r \times N}$ arriving at the BS can be expressed as:

$$\mathbf{Y}_S = \mathbf{G}\mathbf{X} + \mathbf{Z}_S, \quad (3)$$

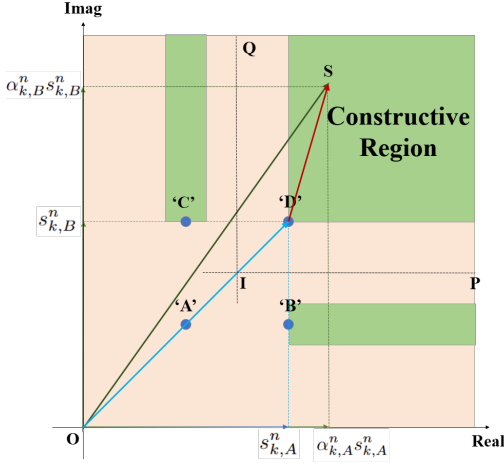


Fig. 2. Geometric illustration of the CI metric for QAM.

where $\mathbf{Z}_S \in \mathbb{C}^{N_r \times N}$ denotes a complex AWGN matrix with variance σ_S^2 . Meanwhile, the target response matrix (TRM) $\mathbf{G} \in \mathbb{C}^{N_r \times N_t}$ in (3) is given by [6]:

$$\mathbf{G} = \sum_{i=1}^{N_S} \alpha_i \mathbf{b}(\theta_i) \mathbf{a}^H(\theta_i). \quad (4)$$

Here, θ_i and α_i represent the angle and reflection coefficient of the i -th point target, respectively. $\mathbf{a}(\theta_i) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{b}(\theta_i) \in \mathbb{C}^{N_r \times 1}$ are the steering vectors of the transmit and receive antennas at the BS, respectively.

The estimation accuracy of the TRM $\mathbf{G}$ is a key performance metric for radar sensing. Therefore, the CRB for estimating $\mathbf{G}$ is commonly adopted as the sensing-performance metric, which can be further characterized via the transmitted signal matrix $\mathbf{X}$ as [6]:

$$\begin{aligned} \text{CRB}(\mathbf{G}) &= \sigma_S^2 N_r \text{tr} [(\mathbf{X}\mathbf{X}^H)^{-1}] \\ &= \frac{1}{2} \sigma_S^2 N_r \text{tr} [(\mathbf{X}_{\text{real}} \mathbf{X}_{\text{real}}^T)^{-1}]. \end{aligned} \quad (5)$$

Here $\mathbf{X}_{\text{real}} = \begin{bmatrix} \Re(\mathbf{X}) & -\Im(\mathbf{X}) \\ \Im(\mathbf{X}) & \Re(\mathbf{X}) \end{bmatrix} \in \mathbb{R}^{2N_t \times 2N}$ is the real-valued extension of $\mathbf{X}$. As a result, the optimization problem of $\mathbf{G}$, i.e., $\min_{\mathbf{G}} \text{CRB}(\mathbf{G})$, becomes:

$$\min_{\mathbf{X}_{\text{real}}} \text{tr} [(\mathbf{X}_{\text{real}} \mathbf{X}_{\text{real}}^T)^{-1}]. \quad (6)$$

B. Constructive Interference

As an effective technique to exploit the information of the data symbols, CI refers to interference that pushes the received signal away from the decision boundaries of the intended constellation point, thereby contributing to the useful signal power [3].

To intuitively illustrate this concept, Fig. 2 presents the first quadrant of a 16-QAM constellation diagram. Assume that $\mathbf{OD}$ is the transmitted data symbol for user k at symbol slot n and $\mathbf{OS}$ is the corresponding noiseless received signal. It can be observed that when $\mathbf{OS}$ falls into the green shaded constructive region, its distance to the decision boundaries $\mathbf{IP}$

and $\mathbf{IQ}$ is larger than that of the nominal constellation point 'D'. Consequently, the same noise is less likely to cause a detection error, which is equivalent to increasing the received signal-to-interference-plus-noise ratio (SINR) [4].

Meanwhile, it should be noted that only the outer constellation points can exploit CI, since the interference is less likely to be beneficial for the inner constellation points [4]. To be more specific, in Fig. 2 CI exists for the real part of the constellation point type 'B' and the imaginary part of type 'C', while both the real and imaginary parts of the constellation point type 'D' can be exploited.

III. PRECODING DESIGN IN CI-AIDED MIMO-ISAC

A. Problem Formulation of CI-Aided MIMO-ISAC

To integrate CI into the MIMO-ISAC framework, we adopt the symbol-scaling metric for QAM constellations [4]. For user k at symbol slot n , the received signal in (2) can be expressed as:

$$y_{k,n} = \mathbf{h}_k^T \mathbf{W} \mathbf{s}^n + z_{k,n} = \alpha_{k,A}^n \Re(s_k^n) + j \alpha_{k,B}^n \Im(s_k^n) + z_{k,n}, \quad (7)$$

where $\alpha_{k,A}^n, \alpha_{k,B}^n \in \mathbb{R}$ are real-scaling coefficients. The subscripts A and B denote the real and imaginary dimensions, respectively.

By controlling the scaling factors, the noiseless received signal can be steered into the constructive region, thereby enhancing the detection reliability. In particular, we can divide $\alpha_{k,A}^n$ and $\alpha_{k,B}^n$ into two sets, leading to the following CI constraints:

$$\alpha_k^n \geq t, \forall (n, k) \in \mathcal{O}, \quad \alpha_k^n = t, \forall (n, k) \in \mathcal{I}. \quad (8)$$

Here, $\mathcal{O}$ denotes the set of scalars that can exploit CI via scaling, while $\mathcal{I}$ contains those that cannot. The variable t serves as a communication quality-of-service (QoS) threshold, which represents the minimum required level of constructive interference scaling.

Given CI constraints in (8), the per-dimension SINRs of CI-aided precoding can be quantified as:

$$\text{SINR}_{k,n}^{\Re, \text{CI}} = \frac{(\alpha_{k,A}^n)^2 |\Re(s_k^n)|^2}{\sigma_C^2}, \quad (9)$$

$$\text{SINR}_{k,n}^{\Im, \text{CI}} = \frac{(\alpha_{k,B}^n)^2 |\Im(s_k^n)|^2}{\sigma_C^2}. \quad (10)$$

By transforming the multiuser interference from a harmful factor into a useful signal component, CI is capable of improving the communication reliability [3].

However, although CI improves communication reliability, the resulting CI constraints restrict the transmit waveform design and may affect sensing performance. This motivates a joint precoding design that accounts for both communication CI requirements and sensing CRB minimization. Specifically, to convert the geometric CI condition in (8) into linear constraints with respect to the optimization variable, we can construct two coefficient matrices $\mathbf{a}_k^n \in \mathbb{R}^{N_t \times 1}$ and $\mathbf{b}_k^n \in \mathbb{R}^{N_t \times 1}$, such that α_k^n can be expressed as [4]:

$$\alpha_k^n = (\mathbf{a}_k^n)^T \hat{\mathbf{W}} \mathbf{s}_E^n + (\mathbf{b}_k^n)^T \hat{\mathbf{W}} \mathbf{c}_E^n, \quad (11)$$

where $\hat{\mathbf{W}} \in \mathbb{R}^{N_t \times 2K}$, $\mathbf{s}_E^n \in \mathbb{R}^{2K \times 1}$ and $\mathbf{c}_E^n \in \mathbb{R}^{2K \times 1}$ are defined as:

$$\hat{\mathbf{W}} = [\Re(\mathbf{W}) \quad -\Im(\mathbf{W})], \mathbf{s}_E^n = \begin{bmatrix} \Re(\mathbf{s}^n) \\ \Im(\mathbf{s}^n) \end{bmatrix}, \mathbf{c}_E^n = \begin{bmatrix} \Im(\mathbf{s}^n) \\ -\Re(\mathbf{s}^n) \end{bmatrix}. \quad (12)$$

Meanwhile, the transmit power constraint can be equivalently rewritten as:

$$\sum_{n=1}^N \|\mathbf{W}\mathbf{s}^n\|_2^2 = \sum_{n=1}^N \left(\|\hat{\mathbf{W}}\mathbf{s}_E^n\|_2^2 + \|\hat{\mathbf{W}}\mathbf{c}_E^n\|_2^2 \right) \leq Np_0. \quad (13)$$

To proceed, by introducing matrices $\mathbf{S}_E = [\mathbf{s}_E^1, \mathbf{s}_E^2, \dots, \mathbf{s}_E^N]$ and $\mathbf{C}_E = [\mathbf{c}_E^1, \mathbf{c}_E^2, \dots, \mathbf{c}_E^N]$, it is straightforward to verify that $\mathbf{X}_{\text{real}} = \begin{bmatrix} \mathbf{W}\mathbf{S}_E & -\mathbf{W}\mathbf{C}_E \\ \hat{\mathbf{W}}\mathbf{C}_E & \hat{\mathbf{W}}\mathbf{S}_E \end{bmatrix} = \hat{\mathbf{W}}_{\text{diag}} \mathbf{U}$, where matrices $\hat{\mathbf{W}}_{\text{diag}} \triangleq \begin{bmatrix} \hat{\mathbf{W}} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{W}} \end{bmatrix}$ and $\mathbf{U} \triangleq \begin{bmatrix} \mathbf{S}_E & -\mathbf{C}_E \\ \mathbf{C}_E & \mathbf{S}_E \end{bmatrix}$. Consequently, by substituting this relationship into (6), the proposed optimization problem, which aims to minimize the sensing CRB under communication CI constraints, can be formulated as:

$$\begin{aligned} \mathcal{P}_1^{\text{QAM}} : \min_{\hat{\mathbf{W}}} \quad & \text{tr} \left[(\hat{\mathbf{W}}_{\text{diag}} \mathbf{U} \mathbf{U}^T \hat{\mathbf{W}}_{\text{diag}}^T)^{-1} \right] \\ \text{s.t.} \quad & (\mathbf{a}_k^n)^T \hat{\mathbf{W}} \mathbf{s}_E^n + (\mathbf{b}_k^n)^T \hat{\mathbf{W}} \mathbf{c}_E^n \geq t, \quad \forall (n, k) \in \mathcal{O}, \\ & (\mathbf{a}_k^n)^T \hat{\mathbf{W}} \mathbf{s}_E^n + (\mathbf{b}_k^n)^T \hat{\mathbf{W}} \mathbf{c}_E^n = t, \quad \forall (n, k) \in \mathcal{I}, \\ & \sum_{n=1}^N \left(\|\hat{\mathbf{W}} \mathbf{s}_E^n\|_2^2 + \|\hat{\mathbf{W}} \mathbf{c}_E^n\|_2^2 \right) \leq Np_0. \end{aligned} \quad (14)$$

A major difficulty in solving (14) lies in the non-convexity of the objective function. This structure motivates the use of SCA, which iteratively approximates the non-convex objective by a tractable surrogate.

B. SCA Algorithm

To tackle the non-convexity in $\mathcal{P}_1^{\text{QAM}}$, we first denote its objective as $f(\hat{\mathbf{W}}) = \text{tr} \left[(\hat{\mathbf{W}}_{\text{diag}} \mathbf{U} \mathbf{U}^T \hat{\mathbf{W}}_{\text{diag}}^T)^{-1} \right]$. At each SCA iteration, we can replace $f(\hat{\mathbf{W}})$ with its first-order Taylor expansion around a given point $\hat{\mathbf{W}}_i$:

$$f(\hat{\mathbf{W}}) \approx f(\hat{\mathbf{W}}_i) + \text{tr} \left(\nabla f(\hat{\mathbf{W}}_i)^T (\hat{\mathbf{W}} - \hat{\mathbf{W}}_i) \right), \quad (15)$$

where the gradient $\nabla f(\hat{\mathbf{W}}_i)$ can be calculated as:

$$\begin{aligned} \nabla f(\hat{\mathbf{W}}_i) = & -2 \left([\mathbf{U} \mathbf{X}_i^T (\mathbf{X}_i \mathbf{X}_i^T)^{-2}]_{1:2K, 1:N_t} \right. \\ & \left. + [\mathbf{U} \mathbf{X}_i^T (\mathbf{X}_i \mathbf{X}_i^T)^{-2}]_{2K+1:4K, N_t+1:2N_t} \right)^T, \end{aligned} \quad (16)$$

with $\mathbf{X}_i = \hat{\mathbf{W}}_{i,\text{diag}} \mathbf{U}$. By this approximation, the non-convex problem $\mathcal{P}_1^{\text{QAM}}$ in (14) can be relaxed into a series of convex sub-problems. Specifically, at the $(i+1)$ -th iteration, the relaxed convex optimization sub-problem corresponds to:

$$\begin{aligned} \mathcal{P}_2^{\text{QAM}} : \min_{\hat{\mathbf{W}}} \quad & \text{tr} \left(\nabla f(\hat{\mathbf{W}}_i)^T (\hat{\mathbf{W}} - \hat{\mathbf{W}}_i) \right) \\ \text{s.t.} \quad & (\mathbf{a}_k^n)^T \hat{\mathbf{W}} \mathbf{s}_E^n + (\mathbf{b}_k^n)^T \hat{\mathbf{W}} \mathbf{c}_E^n \geq t, \quad \forall (n, k) \in \mathcal{O}, \\ & (\mathbf{a}_k^n)^T \hat{\mathbf{W}} \mathbf{s}_E^n + (\mathbf{b}_k^n)^T \hat{\mathbf{W}} \mathbf{c}_E^n = t, \quad \forall (n, k) \in \mathcal{I}, \\ & \sum_{n=1}^N \left(\|\hat{\mathbf{W}} \mathbf{s}_E^n\|_2^2 + \|\hat{\mathbf{W}} \mathbf{c}_E^n\|_2^2 \right) \leq Np_0. \end{aligned} \quad (17)$$

Algorithm 1 SCA Algorithm for $\mathcal{P}_1^{\text{QAM}}$

Input: $\mathbf{H}$, $\mathbf{S}$, t , tolerance ϵ , max iterations $i_{\max}$.

Output: $\mathbf{W}$

- 1: Initialize with RZF as $\hat{\mathbf{W}}_{-1}$ and solve $\mathcal{P}_2^{\text{QAM}}$ to obtain initial feasible point $\hat{\mathbf{W}}_0 \in \mathcal{Q}$, set $i = 0$.
 - 2: **repeat**
 - 3: compute gradient $\nabla f(\hat{\mathbf{W}}_i)$ using equation (16).
 - 4: solve the convex sub-problem $\mathcal{P}_2^{\text{QAM}}$ to obtain $\hat{\mathbf{W}}^*$.
 - 5: update solution by $\hat{\mathbf{W}}_{i+1} = \hat{\mathbf{W}}_i + d(\hat{\mathbf{W}}^* - \hat{\mathbf{W}}_i)$.
 - 6: $i = i + 1$.
 - 7: **until** $\|\hat{\mathbf{W}}_i - \hat{\mathbf{W}}_{i-1}\| \leq \epsilon$ **or** $i = i_{\max}$
 - 8: Recover complex precoder $\mathbf{W}$ according to (19).
-

Here $\hat{\mathbf{W}}_i$ serves as the optimal solution at the i -th iterate, which belongs to the convex feasible set $\mathcal{Q}$, and the threshold t can be obtained via an optimization problem as in [4].

The sub-problem $\mathcal{P}_2^{\text{QAM}}$ in (17) can be solved efficiently by standard convex optimization tools, yielding the optimal solution $\hat{\mathbf{W}}^* \in \mathcal{Q}$. Though approximation errors will occur in the implementation of SCA due to the first-order Taylor expansion, the term $(\hat{\mathbf{W}}^* - \hat{\mathbf{W}}_i)$ yields a descent direction for each iteration. By letting the variable move along the descent direction with a stepsize $d \in (0, 1)$ determined by exact line search, the update of $\hat{\mathbf{W}}_{i+1}$ follows:

$$\hat{\mathbf{W}}_{i+1} = \hat{\mathbf{W}}_i + d(\hat{\mathbf{W}}^* - \hat{\mathbf{W}}_i) = (1-d)\hat{\mathbf{W}}_i + d\hat{\mathbf{W}}^*. \quad (18)$$

Since both $\hat{\mathbf{W}}_i$ and $\hat{\mathbf{W}}^*$ belong to $\mathcal{Q}$, the convexity of $\mathcal{Q}$ guarantees that $\hat{\mathbf{W}}_{i+1} \in \mathcal{Q}$, thereby yielding a sub-optimal solution to problem (14). After obtaining $\hat{\mathbf{W}}$, the original complex precoding matrix $\mathbf{W}$ can be recovered by:

$$\mathbf{W} = \hat{\mathbf{W}}[:, 1:K] - j \hat{\mathbf{W}}[:, K+1:2K] \in \mathbb{C}^{N_t \times K}. \quad (19)$$

The overall SCA procedure is summarized in Algorithm 1.

IV. EFFICIENCY IMPROVEMENT BASED ON THE DUAL PROBLEM FORMULATION

To further reduce computational complexity, we present the *Lagrange* dual of $\mathcal{P}_2^{\text{QAM}}$ with fewer constraints. In particular, the *Lagrangian* of $\mathcal{P}_2^{\text{QAM}}$ can be constructed as:

$$\begin{aligned} \mathcal{L}(\hat{\mathbf{W}}, \delta_{k,n}, \lambda) = & \text{tr}(\Psi^T \hat{\mathbf{W}}) + \sum_{k,n} \delta_{k,n} \left(t - \text{tr}(\mathbf{M}_{k,n}^T \hat{\mathbf{W}}) \right) \\ & + \lambda \left(\text{tr}(\hat{\mathbf{W}} \mathbf{D} \hat{\mathbf{W}}^T) - Np_0 \right), \end{aligned} \quad (20)$$

with $\Psi \triangleq \nabla f(\hat{\mathbf{W}}_i)$, $\mathbf{M}_{k,n}^T \triangleq \mathbf{s}_E^n (\mathbf{a}_k^n)^T + \mathbf{c}_E^n (\mathbf{b}_k^n)^T$, and $\mathbf{D} \triangleq \mathbf{S}_E \mathbf{S}_E^T + \mathbf{C}_E \mathbf{C}_E^T$. λ and $\delta_{k,n}$ represent the non-negative dual variables. Accordingly, with $\mathbf{M}(\delta) \triangleq \sum_{k,n} \delta_{k,n} \mathbf{M}_{k,n}$, the KKT conditions for optimality of $\mathcal{P}_2^{\text{QAM}}$ can be stated as:

$$\frac{\partial \mathcal{L}}{\partial \hat{\mathbf{W}}} = -(\mathbf{M}(\delta) - \Psi) + 2\lambda \hat{\mathbf{W}} \mathbf{D} = \mathbf{0}, \quad (21a)$$

$$\delta_{k,n} \left(t - \text{tr}(\mathbf{M}_{k,n}^T \hat{\mathbf{W}}) \right) = 0, \quad \forall (k, n) \in \mathcal{O}, \quad (21b)$$

$$t - \text{tr}(\mathbf{M}_{k,n}^T \hat{\mathbf{W}}) = 0, \quad \forall (k, n) \in \mathcal{I}, \quad (21c)$$

$$\lambda \left(\text{tr}(\hat{\mathbf{W}} \mathbf{D} \hat{\mathbf{W}}^T) - Np_0 \right) = 0. \quad (21d)$$

From the KKT conditions, we can conclude that $\lambda > 0$ by contradiction. Otherwise, no $\delta_{k,n}$ can satisfy (21a). This indicates that the total power constraint is active upon reaching optimality, i.e., $\text{tr}(\hat{\mathbf{W}}\mathbf{D}\hat{\mathbf{W}}^T) = Np_0$. Given that the block length N is typically larger than the number of users K , $\mathbf{D}$ is thus full-rank and invertible [4]. This allows us to derive a closed-form expression for the optimal precoding matrix:

$$\hat{\mathbf{W}}^*(\delta, \lambda) = \frac{1}{2\lambda}(\mathbf{M}(\delta) - \Psi)\mathbf{D}^{-1}. \quad (22)$$

To proceed, by substituting (22) into the power equality in (21d), we can obtain the following expression for λ :

$$\lambda^*(\delta) = \sqrt{\frac{\text{tr}((\mathbf{M}(\delta) - \Psi)\mathbf{D}^{-1}(\mathbf{M}(\delta) - \Psi)^T)}{4Np_0}}. \quad (23)$$

Moreover, since the convex sub-problem $\mathcal{P}_2^{\text{QAM}}$ satisfies Slater's condition, its optimal solution can be obtained by solving the Lagrange dual problem [7]:

$$\max_{\delta_{k,n}, \lambda} \min_{\hat{\mathbf{W}}} \mathcal{L}(\hat{\mathbf{W}}, \delta_{k,n}, \lambda). \quad (24)$$

Consequently, via (22) and (23), the final dual problem of $\mathcal{P}_2^{\text{QAM}}$, which focuses solely on $\delta_{k,n}$, is formulated as:

$$\begin{aligned} \mathcal{P}_3^{\text{Dual}} : \min_{\delta_{k,n}} & \sqrt{Np_0} \sqrt{\text{tr}((\mathbf{M}(\delta) - \Psi)\mathbf{D}^{-1}(\mathbf{M}(\delta) - \Psi)^T)} \\ & - t \sum_{k,n} \delta_{k,n} \\ \text{s.t. } & \delta_{k,n} \geq 0, \quad \forall (k, n) \in \mathcal{O}. \end{aligned} \quad (25)$$

Given that $\delta_{k,n}$ for $(k, n) \in \mathcal{I}$ are unconstrained and the block-level power constraint is no longer required, $\mathcal{P}_3^{\text{Dual}}$ corresponds to a simplified problem. Compared with the primal sub-problem $\mathcal{P}_2^{\text{QAM}}$, which deals with more complex cones, this structural difference makes solving $\mathcal{P}_3^{\text{Dual}}$ more efficient, leading to much lower computational complexity [8].

The dual SCA algorithm adopts the same iterative framework as Algorithm 1. Specifically, each iteration involves solving the convex sub-problem $\mathcal{P}_3^{\text{Dual}}$ to derive the dual variables $\delta_{k,n}$, followed by the closed-form recovery of the optimal precoding matrix $\hat{\mathbf{W}}^*$ via (22). Motivated by the above analysis, we propose Algorithm 2 as a computationally attractive alternative for obtaining a sub-optimal solution to (14). To further reduce computational complexity, the communication threshold t can be predicted by lightweight neural networks while maintaining competitive system performance.

V. NUMERICAL RESULTS

A. Simulation Setup

In this section, we numerically evaluate the performance of the proposed algorithms. Throughout the simulations, the transmit power budget per symbol slot is set as $p_0 = 1$, and 16-QAM modulation is employed. The following abbreviations are used throughout this section:

- **CI-BLP**: The CI-BLP precoding proposed in [4].
- **CI-SCA Exact**: The proposed CI-SCA precoding based on Algorithm 1.

Algorithm 2 Dual SCA Algorithm for $\mathcal{P}_1^{\text{QAM}}$

Input: $\mathbf{H}$, $\mathbf{S}$, t , tolerance ϵ , max iterations $i_{\max}$.

Output: $\mathbf{W}$

- 1: Initialize with RZF as $\hat{\mathbf{W}}_{-1}$ and solve $\mathcal{P}_3^{\text{Dual}}$ to obtain $\delta_{k,n}$, then compute $\hat{\mathbf{W}}_0$ via (22), set $i = 0$.
- 2: **repeat**
- 3: compute gradient $\nabla f(\hat{\mathbf{W}}_i)$ using equation (16).
- 4: solve the convex sub-problem $\mathcal{P}_3^{\text{Dual}}$ to obtain $\delta_{k,n}$.
- 5: compute optimal $\hat{\mathbf{W}}^*$ according to equation (22).
- 6: update solution by $\hat{\mathbf{W}}_{i+1} = \hat{\mathbf{W}}_i + d(\hat{\mathbf{W}}^* - \hat{\mathbf{W}}_i)$.
- 7: $i = i + 1$.
- 8: **until** $\|\hat{\mathbf{W}}_i - \hat{\mathbf{W}}_{i-1}\| \leq \epsilon$ **or** $i = i_{\max}$
- 9: Recover complex precoder $\mathbf{W}$ according to (19).

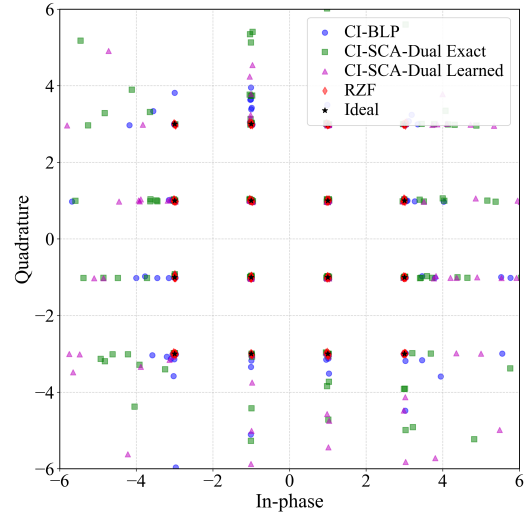


Fig. 3. Received constellation diagrams of different schemes with $K = 12$, $N = 15$.

- **CI-SCA-Dual Exact**: The proposed dual SCA precoding based on Algorithm 2.
- **CI-SCA-Dual Learned**: The proposed dual SCA precoding, using lightweight predictive neural networks.

B. Performance Comparison

Fig. 3 compares the received constellation diagrams of different precoding methods. As can be seen, the received noisy symbols under RZF precoding are located near the nominal constellation points. In contrast, for CI-based precoding, the received noisy symbols are pushed away from the decision boundaries, which confirms the effectiveness of CI precoding.

Fig. 4 depicts the SER performance of different precoding schemes. It is observed that the proposed CI-SCA-Dual performs slightly worse than CI-BLP, since CI-SCA-Dual prioritizes sensing performance. Since noise dominates at low SNR, RZF precoding exhibits better SER performance owing to its regularized structure. Nevertheless, as the SNR increases, both CI-based precoding approaches achieve improved performance over RZF.

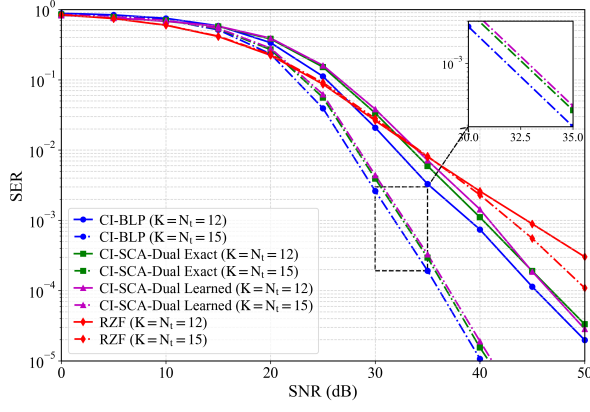


Fig. 4. SER performance of different schemes with $N = 15$.

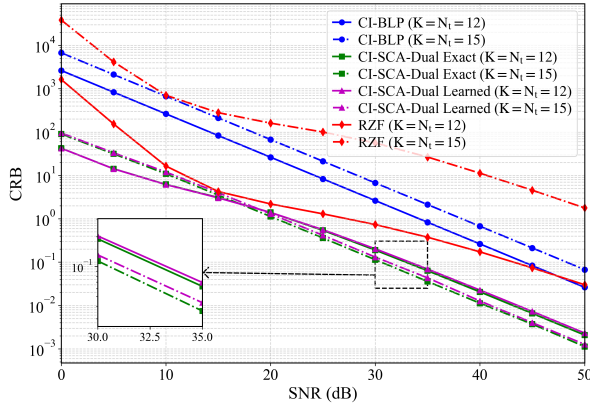


Fig. 5. CRB of different schemes with $N = 15$.

Fig. 5 compares the radar estimation CRB of different precoding methods. Among them, CI-BLP precoding exhibits the worst performance, as its maximization of t tends to concentrate transmit energy on communication. In contrast, the proposed CI-SCA-Dual precoding achieves the best sensing accuracy across the entire SNR range via joint optimization.

Fig. 6 illustrates the complexity of different schemes in terms of the execution time. For fairness, all convex optimization problems are solved using the Mosek solver. As expected, CI-SCA precoding incurs higher complexity owing to SCA iterations, representing a typical complexity-performance trade-off. Meanwhile, it can be observed that solving the dual problem is more efficient than solving the original problem. Furthermore, the machine learning method yields additional complexity reduction, demonstrating its viability.

VI. CONCLUSION

In this paper, we proposed a CI-BLP design for QAM-modulated MIMO-ISAC systems. The formulated non-convex problem minimized the sensing CRB subject to communication CI constraints. A sub-optimal solution was obtained using SCA, and a *Lagrange*-dual-based implementation was further

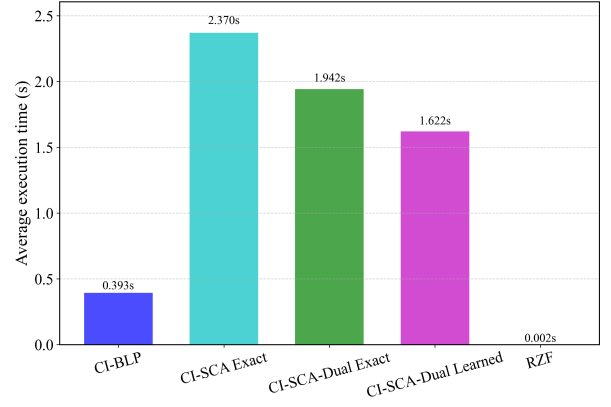


Fig. 6. Execution time of different schemes with $K = N_t = 12$, $N = 15$.

developed to improve computational efficiency. Numerical results demonstrated that the proposed CI-SCA-Dual precoding achieved significant improvement in ISAC performance with limited computational complexity increment.

ACKNOWLEDGMENT

This work was supported in part by the National Key R&D Program of China under Grants No.2023YFC2205501, in part by National Science and Technology Major Projects of China under Grant 2025ZD1301800, and in part by National Natural Science Foundation of China under Grants No.62371124.

REFERENCES

- [1] F. Liu, C. Masouros, A. P. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: applications, state-of-the-art, and the road ahead," *IEEE Trans. Commun.*, vol. 68, no. 6, pp. 3834-3862, Jun. 2020.
- [2] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439-441, May 1983.
- [3] A. Li et al., "A tutorial on interference exploitation via symbol-level precoding: overview, state-of-the-art and future directions," *IEEE Commun. Surveys Tuts.*, vol. 22, no. 2, pp. 796-839, Secondquarter 2020.
- [4] A. Li, C. Shen, X. Liao, C. Masouros, and A. L. Swindlehurst, "Practical interference exploitation precoding without symbol-by-symbol optimization: a block-level approach," *IEEE Trans. Wireless Commun.*, vol. 22, no. 6, pp. 3982-3996, Jun. 2023.
- [5] Y. Wang, X. Hu, A. Li, K.-K. Wong, and K. Yang, "CI-based block-level precoding for ISAC system," in *2024 16th International Conference on Wireless Communications and Signal Processing (WCSP)*, Hefei, China, 2024, pp. 869-874.
- [6] F. Liu, S. Lu, and Z. Chen, "MIMO-ISAC precoding design toward random signals," *Journal of Radars*, vol. 14, no. 4, pp. 781-796, 2025.
- [7] S. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge: Cambridge University Press, 2004.
- [8] F. Alizadeh, and D. Goldfarb, "Second-order cone programming," *Math. Program.*, Ser B, vol. 95, pp. 3-51, 2003.
- [9] Y. Zhao et al., "Metasurface router for near-field multi-user low-interference access driven by structured wave," *IEEE Trans. Veh. Technol.*, early access, 2026, doi: 10.1109/TVT.2026.3728032.
- [10] Y. Zhao, X. Jiang, Z. Liu, Y. L. Guan, and C. Yuen, "Azimuth-dependent secure transmission with orbital angular momentum directional modulation," *IEEE Trans. Veh. Technol.*, early access, 2026, pp. 1-6, doi: 10.1109/TVT.2026.3719765.
- [11] X. Feng et al., "OFDM-based waveform design for MIMO DFRC systems with reduced range sidelobes: A majorization-minimization approach," *IEEE Trans. Veh. Technol.*, vol. 74, no. 3, pp. 4582-4595, Mar. 2025.