A Massive MIMO Sampling Detection Strategy Based on Denoising Diffusion Model

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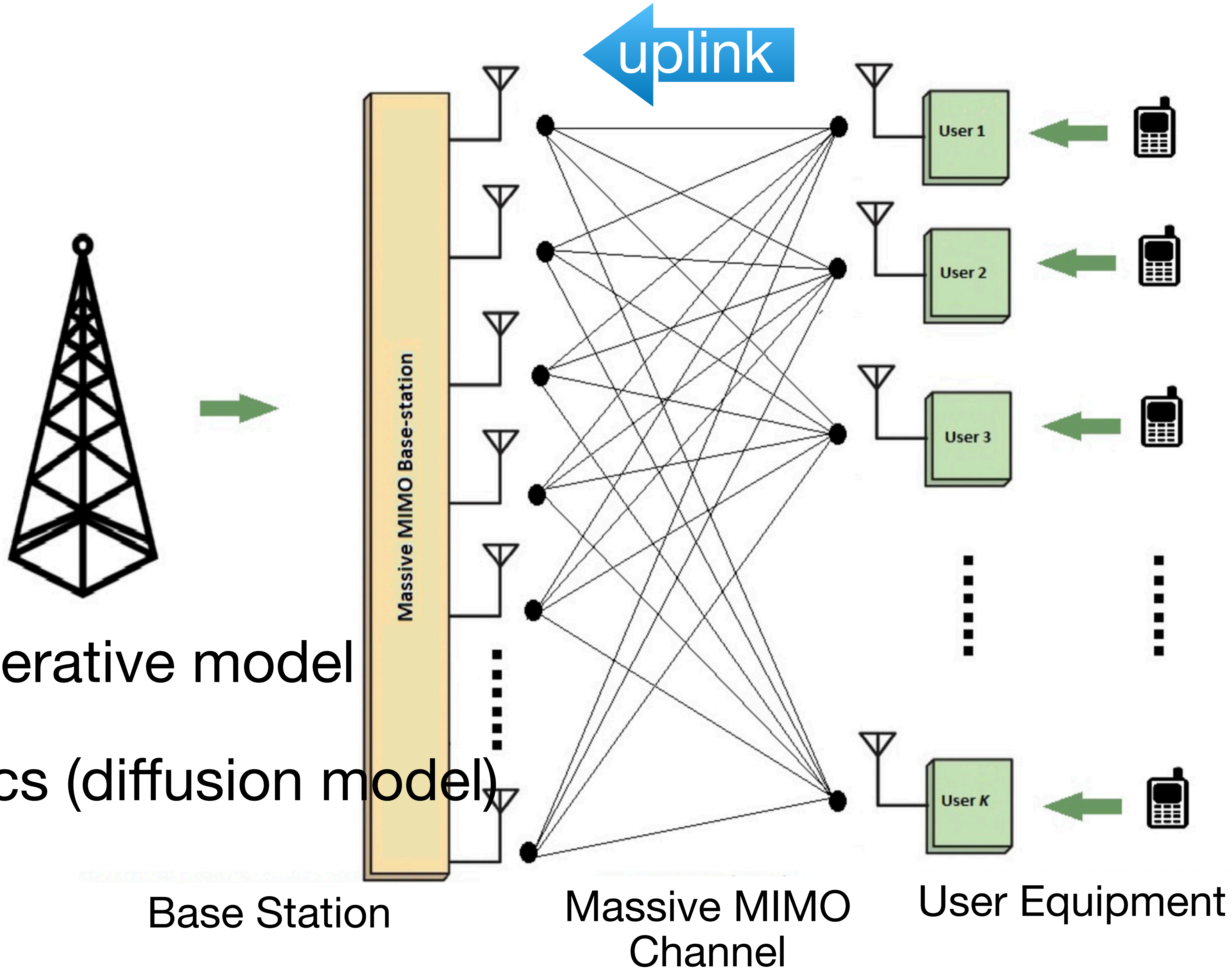
Outlines

- **Introduction**
- **Score-based Massive MIMO Detection**
- **Approximate Diffusion Detection Strategy**
- **Simulation results**
- **Conclusion**

Introduction

- Massive MIMO Detection
  - On the uplink side



- ***Diffusion model***/score-based generative model
  - non-equilibrium statistical physics (diffusion model)
  - Denoising score matching
  - Denoising diffusion probabilistic model (DDPM)
  - Stochastic differential equation (SDE) for unification

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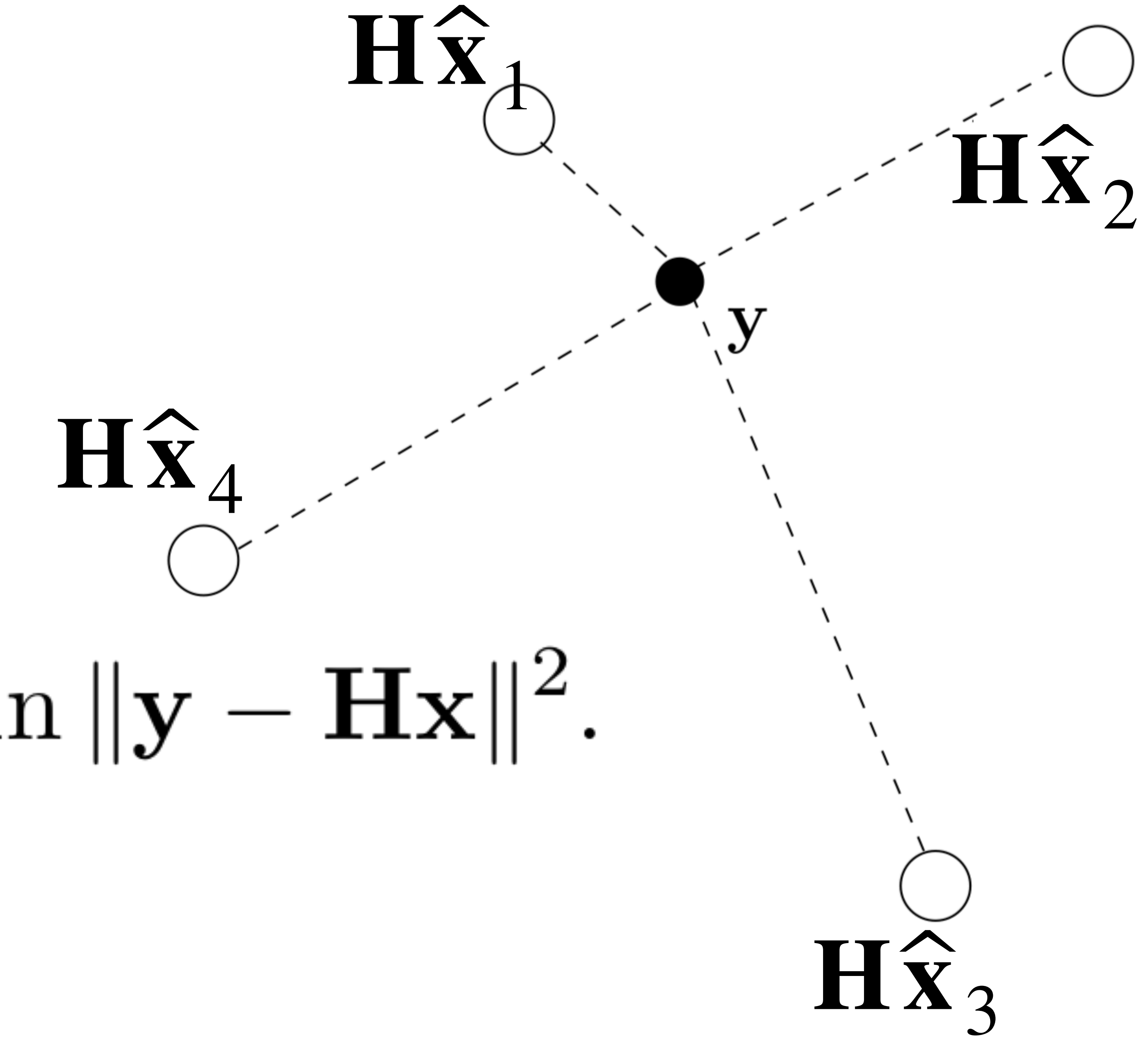
Score-based Massive MIMO Detection

- System Model $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$

  $\mathbf{H} \in \mathbb{R}^{N \times K}$ perfectly-known flat Rayleigh fading channel matrix

  $\mathbf{n} \in \mathbb{R}^{N}$ zero-mean additive white Gaussian noise with variance σ_0^2

  $\mathcal{Q} = \{\pm 1, \pm 3, ..., \pm\sqrt{M} - 1\}$ constellation set for M-ary QAM



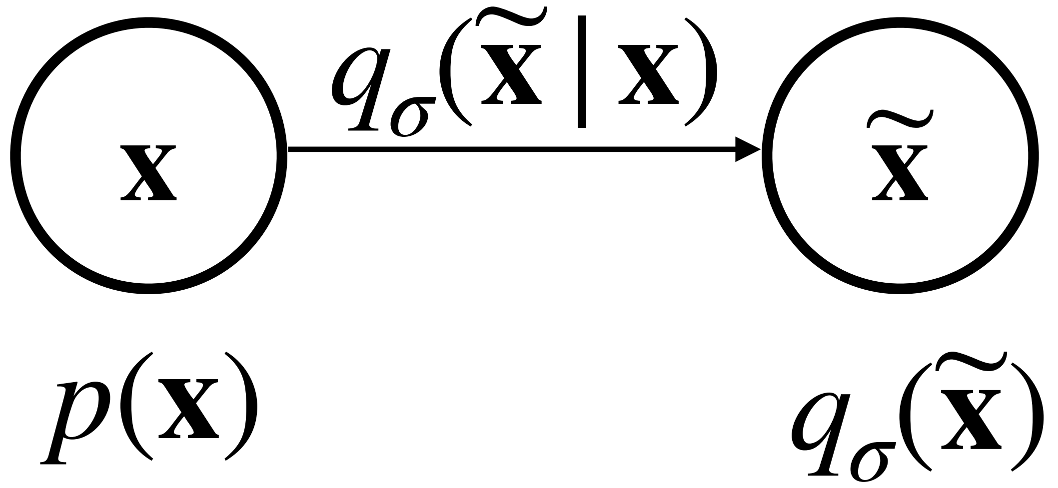
- MAP to ML detection

$$\widehat{\mathbf{x}}_{\mathrm{MAP}} = \arg\max_{\mathbf{x}\in\mathcal{Q}^K} p(\mathbf{x}|\mathbf{y},\mathbf{H}) = \arg\max_{\mathbf{x}\in\mathcal{Q}^K} p(\mathbf{y}-\mathbf{H}\mathbf{x})p(\mathbf{x}).$$

uniform assumption on prior

$$\widehat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\mathbf{x}\in\mathcal{Q}^K} \|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2.$$

Score-based Massive MIMO Detection



- Denoising Score-Matching
  - Definition of *score:* $\mathbf{s}(\mathbf{x}) \triangleq \nabla_{\mathbf{x}} \log p(\mathbf{x})$
  - score matching $\mathbf{s}_\theta(\mathbf{x})$ Approximate $\mathbf{s}(\mathbf{x})$
  - Denoising score matching to perturb: $q_\sigma(\tilde{\mathbf{x}}) = \int p(\mathbf{x}) q_\sigma(\tilde{\mathbf{x}}|\mathbf{x}) d\mathbf{x}$

    With Gaussian kernel (perturbation) $q_\sigma(\tilde{\mathbf{x}}|\mathbf{x}) = \mathcal{N}(\tilde{\mathbf{x}}; \mathbf{x}, \sigma^2\mathbf{I})$
    - Optimization criterion

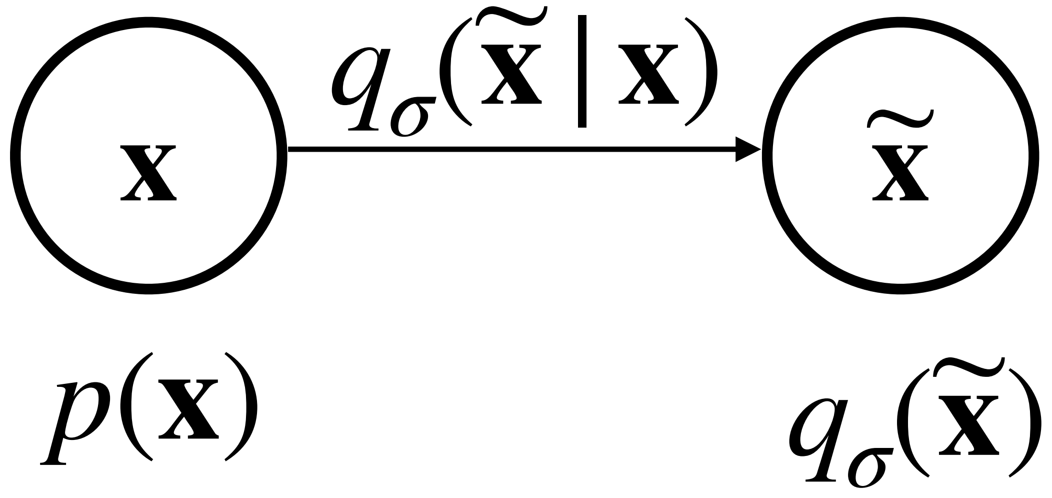
$$\theta = \arg\min_\theta \mathbb{E}_{q_\sigma(\tilde{\mathbf{x}}|\mathbf{x})p(\mathbf{x})}[\|\mathbf{s}_\theta(\tilde{\mathbf{x}}) - \nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}}|\mathbf{x})\|^2]$$

    - As long as the perturbation is small enough:

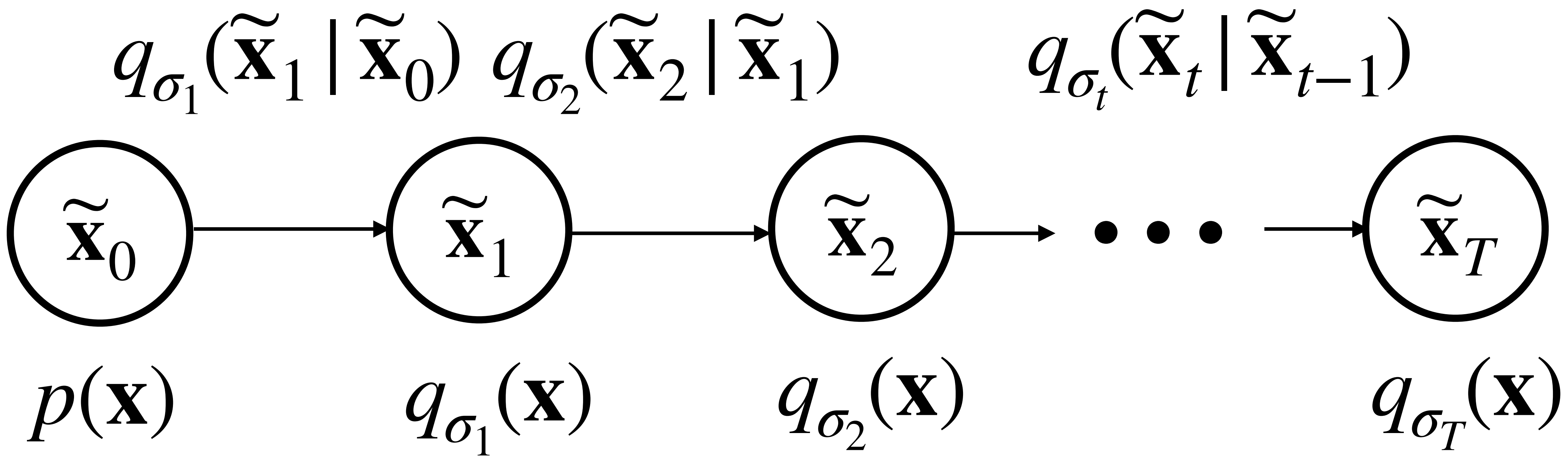
$$\mathbf{s}_\theta^*(\mathbf{x}) = \nabla_{\mathbf{x}} \log q_\sigma(\mathbf{x}) \approx \nabla_{\mathbf{x}} \log p(\mathbf{x})$$

Score-based Massive MIMO Detection

- Annealed Langevin Strategy



irregular fluctuation
estimation inaccuracy
slow mixing problem

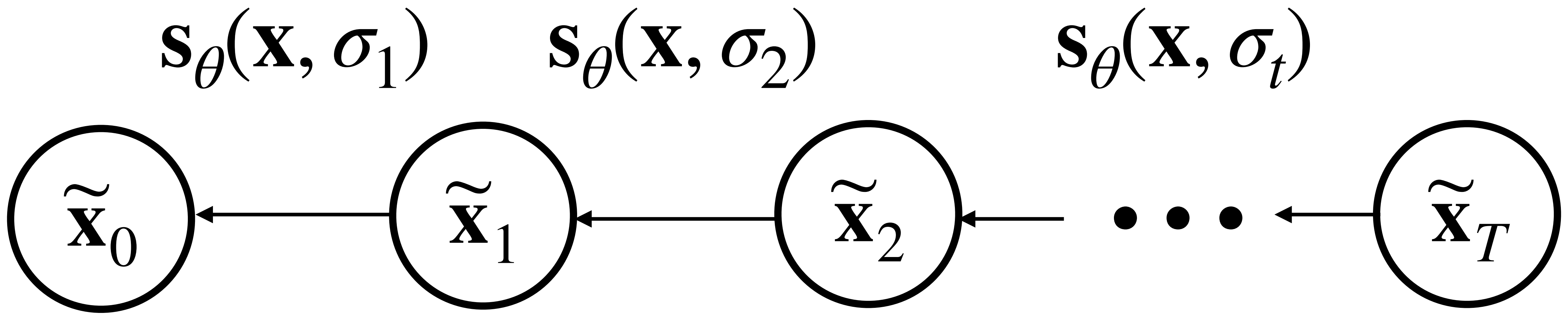


forward diffusion processes

For the Gaussian kernel: $q_{\sigma_t}(\tilde{\mathbf{x}}_t|\mathbf{x}) \sim \mathcal{N}(\tilde{\mathbf{x}}_t; \mathbf{x}, \sigma_t^2\mathbf{I})$

Reverse generative process

$$\frac{\sigma_1}{\sigma_2} = \cdots = \frac{\sigma_{T-1}}{\sigma_T} < 1$$



$$\hat{\mathbf{x}}_i = \hat{\mathbf{x}}_{i-1} + \frac{\delta_t}{2}\mathbf{s}_\theta(\hat{\mathbf{x}}_{i-1}, \sigma_t) + \sqrt{\delta_t}\mathbf{w}_i, i = 1, 2, \cdots, L_A$$

$$\hat{\mathbf{x}}_0 = \tilde{\mathbf{x}}_t$$
$$\tilde{\mathbf{x}}_{t-1} = \hat{\mathbf{x}}_{L_A}$$

Score-based Massive MIMO Detection

- Annealed Langevin sampling detection (ALS) [1] $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$
  - *Posterior score* matters
  - Applying the Bayes' rule

$$\nabla_{\widehat{\mathbf{x}}_t} \log p(\widehat{\mathbf{x}}_t|\mathbf{y}) = \nabla_{\widehat{\mathbf{x}}_t} \log p(\widehat{\mathbf{x}}_t) + \nabla_{\widehat{\mathbf{x}}_t} \log p(\mathbf{y}|\widehat{\mathbf{x}}_t)$$

  - SNIPS Method: evaluate the score explicitly
    - Involving an SVD decomposition of the channel matrix $\mathbf{H}$
  - list detection $\mathcal{L} = \{\widehat{\mathbf{x}}^{(1)}, \widehat{\mathbf{x}}^{(2)}, \cdots, \widehat{\mathbf{x}}^{(S)} | \widehat{\mathbf{x}} \sim p(\mathbf{x}|\mathbf{y})\}$

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{L}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

[1] N. Zilberstein, C. Dick, R. Doost-Mohammady, A. Sabharwal, and S. Segarra, "Annealed Langevin dynamics for massive MIMO detection," IEEE Trans. Wireless Commun., vol. 22, no. 6, pp. 3762-3776, 2023.

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Approximate Diffusion Detection Strategy

- **objective**
  - to circumvent the SVD decomposition
  - To find a flexible sampling structure that can be extended to ***deep generative detection network***
- **Main idea**
  - Incorporate with an iterative detection method to sample from an *approximate posterior distribution*

posterior distribution $p(\mathbf{x}\,|\,\mathbf{y})$

Untractable

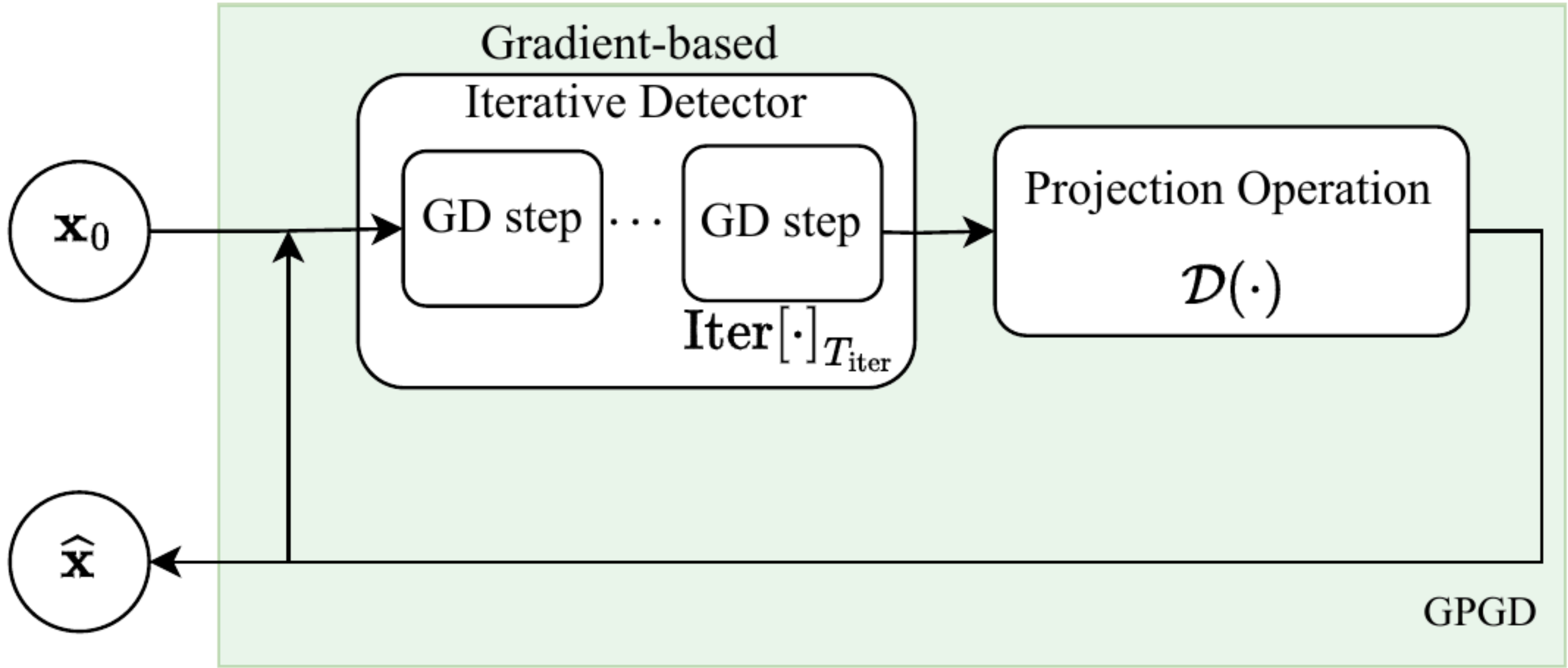
→

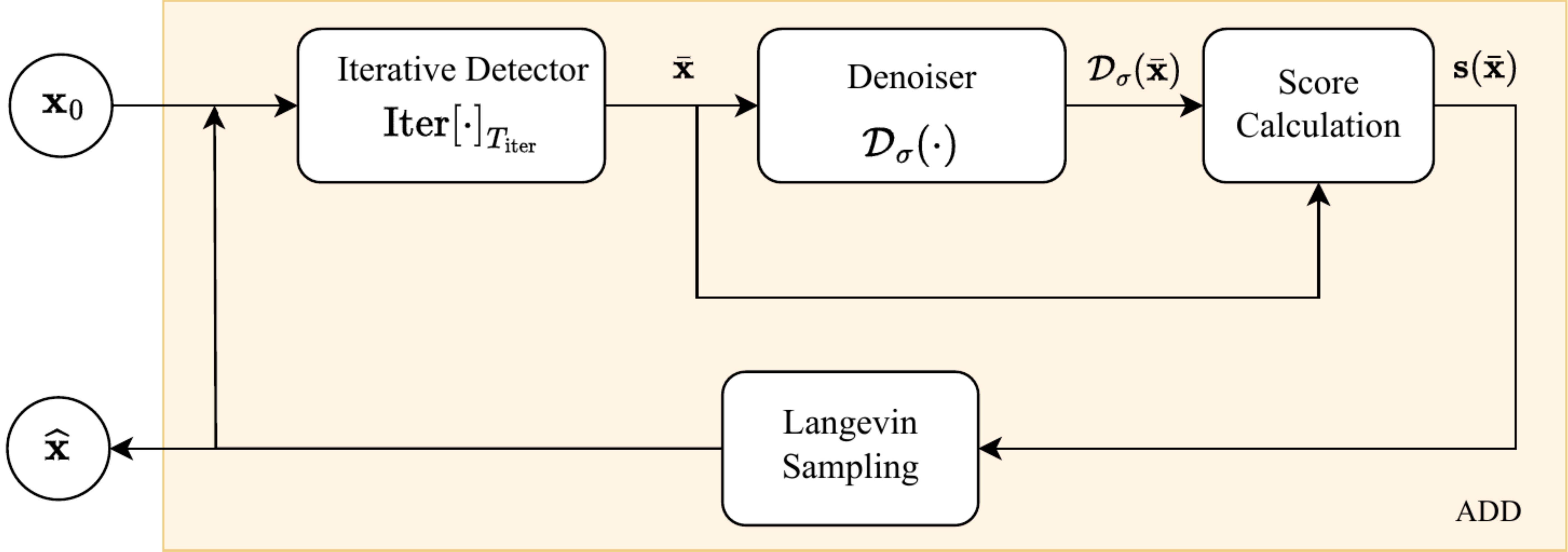
Approximate posterior distribution $\widehat{p}(\mathbf{x}\,|\,\mathbf{y})$

tractable

Approximate Diffusion Detection Strategy

- Inspiration to propose Approximate Diffusion Detection (ADD) Strategy





- Give a direction in the sampling space

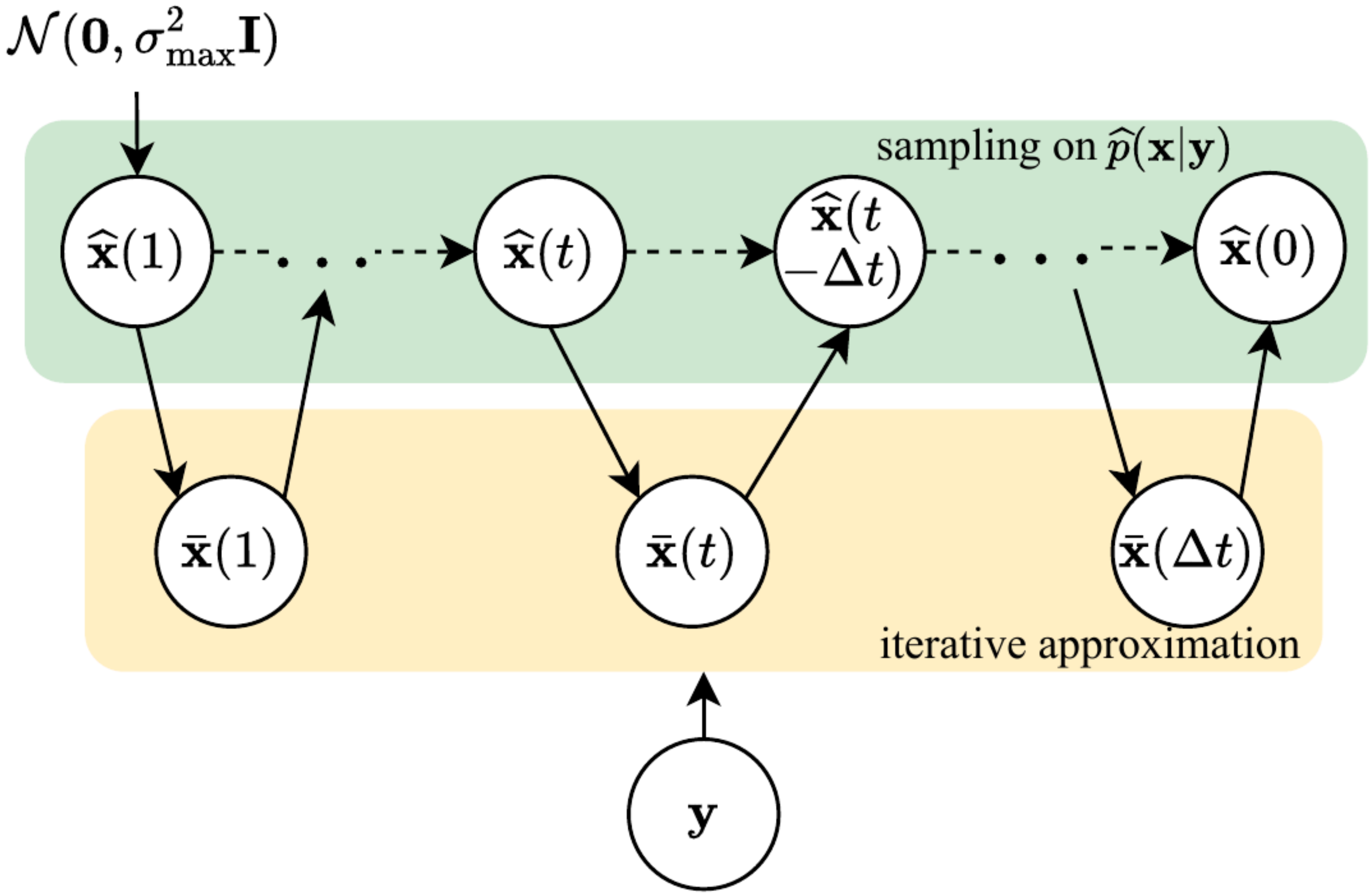
$$\widehat{\mathbf{x}}_i = \widehat{\mathbf{x}}_{i-1} + \frac{\delta_t}{2}\mathbf{s}_\theta(\widehat{\mathbf{x}}_{i-1}, \sigma_t) + \sqrt{\delta_t}\mathbf{w}_i, i = 1, 2, \cdots, L_A$$

Random walk

- Apply a deterministic method stochastically

Approximate Diffusion Detection Strategy

- Structure



- Details on the score calculation
  - The denoisor
    - 1-D Lattice Gaussian Distribution (LGD)

$$p_{\mathcal{Q}}(x_k = \widehat{x}_k; \bar{x}_k, \sigma) \triangleq \frac{1}{Z_{\mathcal{Q}}} \exp\left(\frac{-\|\widehat{x}_k - \bar{x}_k\|^2}{2\sigma^2}\right), \widehat{x}_k \in \mathcal{Q},$$

$$Z_{\mathcal{Q}} = \sum_{\widehat{x}_k \in \mathcal{Q}} \exp\left(\frac{-\|\widehat{x}_k - \bar{x}_k\|^2}{2\sigma^2}\right)$$

  - Tweedie's identity

$$\mathbf{s}(\bar{\mathbf{x}}) = \frac{\mathcal{D}(\bar{\mathbf{x}}) - \bar{\mathbf{x}}}{\sigma^2}$$

Score-based Massive MIMO Detection

- Complexity
  - Take conjugated gradient descent (CGD) as an instance

ADD	ALS
$O(NK^2+T(NK+T_{\text{iter}}K^2+MK))$	$O(NK^2+L_AT(K^2+MK))$
calculation of Hermitian matrix $\mathbf{H}^T\mathbf{H}$	SVD computation

- **Extension to deep generative detection network**

For the Gaussian kernel: $\nabla_{\widetilde{\mathbf{x}}} \log q_\sigma(\widetilde{\mathbf{x}} \mid \mathbf{x}) = -(\widetilde{\mathbf{x}} - \mathbf{x})/\sigma^2$

Training of a denoising score network $\ell(\theta;\sigma) = \mathbb{E}_{p(\mathbf{x})}\mathbb{E}_{\widetilde{\mathbf{x}}\sim q_\sigma(\widetilde{\mathbf{x}}|\mathbf{x})}\left[\left\|\mathbf{s}_\theta(\widetilde{\mathbf{x}},\sigma) + \frac{\widetilde{\mathbf{x}}-\mathbf{x}}{\sigma^2}\right\|^2\right]$

Consider the reparameterization $\widetilde{\mathbf{x}} = \mathbf{x} + \sigma\mathbf{z}$ $\quad \ell(\theta;\sigma) = \mathbb{E}_{p(\mathbf{x})}\mathbb{E}_{\mathbf{z}\sim\mathcal{N}(\mathbf{0},\mathbf{I})}\left[\left\|\mathbf{s}_\theta(\widetilde{\mathbf{x}},\sigma) + \frac{\mathbf{z}}{\sigma}\right\|^2\right]$

unsupervised training

Minimum MSE denoiser $\mathbf{s}(\widetilde{\mathbf{x}},\sigma) = \frac{\mathcal{D}_\sigma(\widetilde{\mathbf{x}}) - \widetilde{\mathbf{x}}}{\sigma^2}$ $\quad \ell(\theta;\sigma) = \mathbb{E}_{p(\mathbf{x})}\mathbb{E}_{\widetilde{\mathbf{x}}\sim q_\sigma(\widetilde{\mathbf{x}}|\mathbf{x})}\left[\|\mathcal{D}_{\theta,\sigma}(\widetilde{\mathbf{x}}) - \mathbf{x}\|^2\right]$

supervised training

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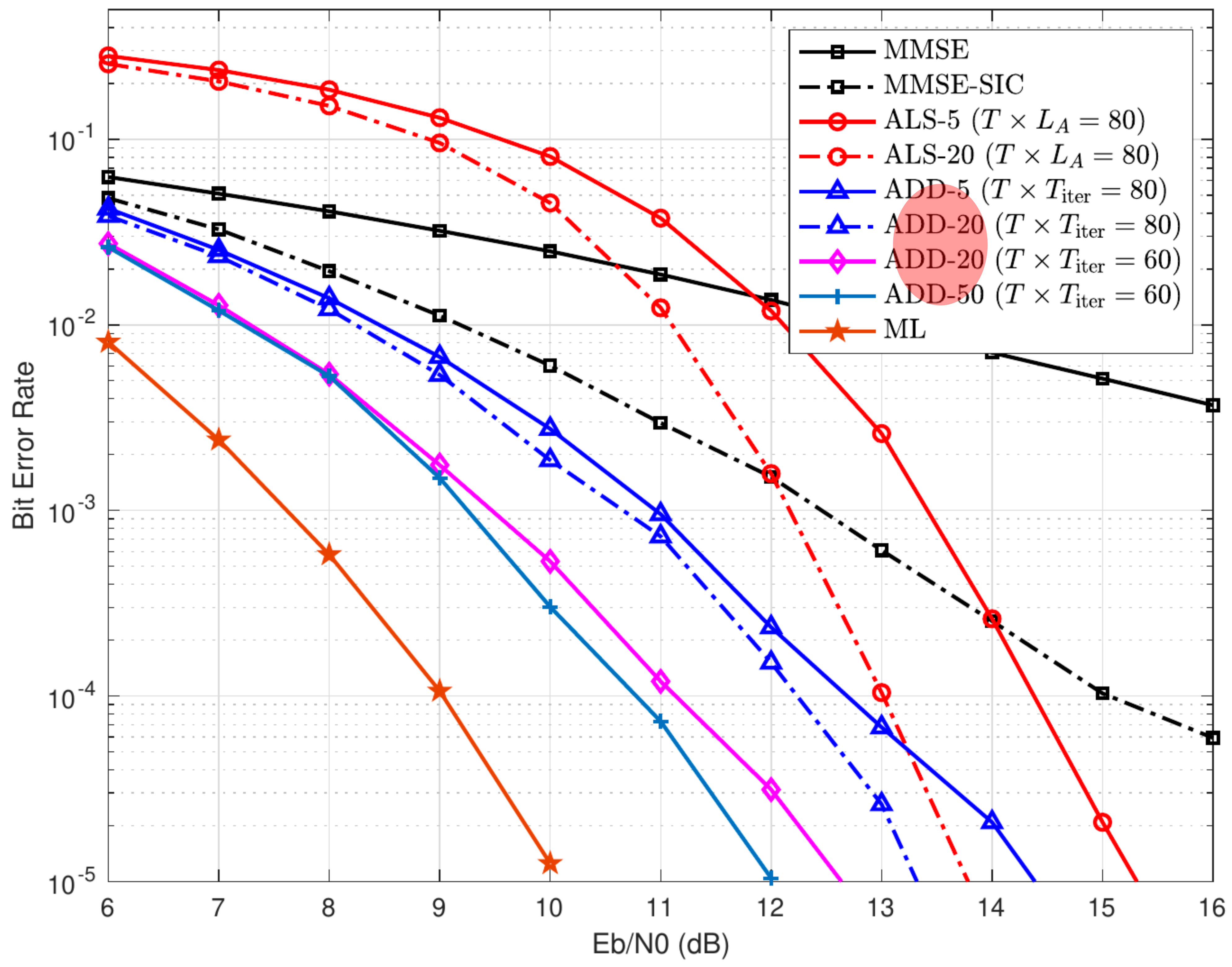
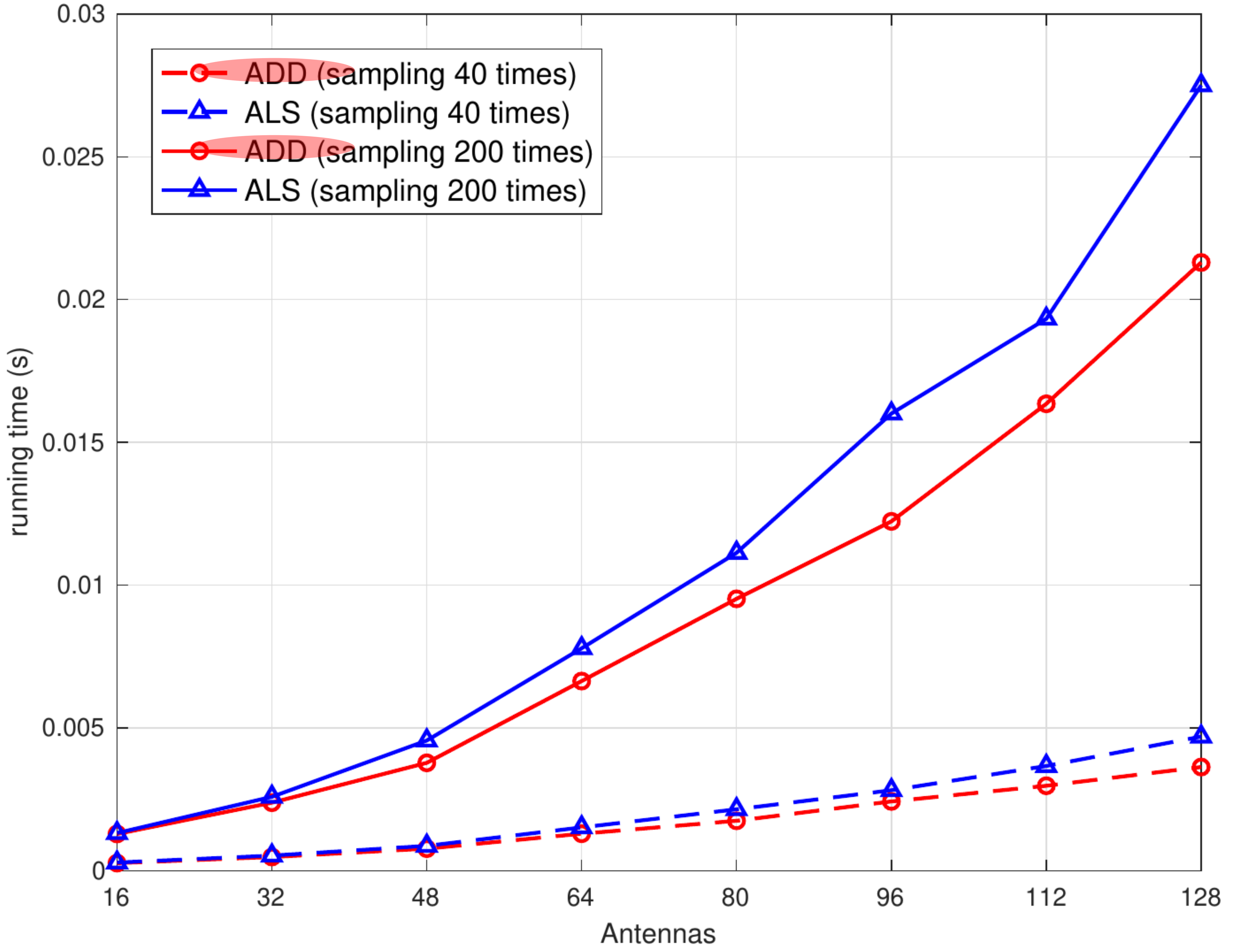
Simulation results



32 × 32 QPSK
Performance



N = K
Running time

A better performance-complexity trade-off

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Conclusion

- Circumventing the SVD calculation of the existing score-based detection
- A strategy that can be applied to other gradient-based methods flexibly
- Applying a deterministic algorithm stochastically

- **Future work**
    - Network structure for the extended ***deep generative detection network***
    - Formulation of the proposal distribution

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Thanks for listening, any questions?

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